

A COMPLEAT
TREATISE
OF
MENSURATION,
IN ALL IT'S
BRANCHES;
CONTAINING
Many New and Necessary
IMPROVEMENTS,

IN

A much more *easy* and *familiar* METHOD
than any hitherto extant.

The Whole adapted not only to be useful
to EXPERIENCED MEASURERS, but
also to YOUNG LEARNERS of the RU-
DIMENTS of *Mensuration*; and may serve
as an EASY INTRODUCTION to several
Parts of the *Mathematicks*.

By J. ROBERTSON,
Teacher of the MATHEMATICS.

LONDON:

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THE P R E F A C E.

 *AMONG the several Branches of the Mathematics taught by me, that of Mensuration has very often occurred; when finding none of the Books on that Subject but what were defective, as also not so methodical as I could have wish'd, I was thereby induced to compose this new System for the Use of my Pupils, which I apprehend will be found not only fully to answer the Title, but likewise an Introduction to the higher Branches of the Mathematics.*

I presume the Reader to be previously acquainted with Addition, Subtraction, Multiplication, Division, and Reduction in whole Numbers, [or Integers] by which he will be able to comprehend with Ease, all that this Book contains.

As an Introduction I have given the Doctrine of Decimal Arithmetic, wherein I have not only handled the Rules commonly given in the Books of Mensuration; but have also fully treated of the Doctrine of Single Repetends (or Circulating Numbers) a Thing so useful in Measuring, that I wonder so many Books on that Subject have been wrote without considering them: For it very often happens, that the Dimensions of Superficies and Solids will not be either 3, 6, or 9 Inches (whose Decimals are Terminate) which are generally the Inches annexed to Feet in the Examples given in the Books of Measuring, when the same is to be wrought by Decimals.

I have purposely omitted the Doctrine of Compound Repetends, they seldom occurring in common Mensuration; but those who are willing to know more of that Subject, I refer them to the late ingenious Mr CUN's Treatise of Fractions, or to a Treatise now in the Press, intituled The Compendious Astronomer, written by my Friend and Acquaintance Mr C. Brent, where the whole Theory of Decimals is amply explained and Demonstrated.

To this I have added the Extraction of the Square and Cube Root, with the Explanation of the common Characters (or Signs) used in Mathematical Books, to shorten the Work.

In the First Part of the Measuring I have fully treated of the Mensuration of all Superficial Figures, as far as a Right Line and a Circle are concerned;

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v

concerned; and herein I have observed the following Order :

First, To give such Definitions as are necessary to conceive the Form of plain Figures.

Secondly, Three Geometrical Problems.

Thirdly, From two Propositions to deduce the Measurement of such Sort of Figures, as most commonly occur to the several Sorts of Artificers concerned in Building; and herein I have shewn how to operate by Duodecimals (commonly called Cross Multiplication) in a more easy Manner than any yet published. And have wrought most of the Examples, both by Duodecimals and Decimals, whereby the Methods may be compared.

And Fourthly, I have given Rules and proper Examples whereby any other Right-lined Figure may be measured; and the Circle I have treated of in a different (though I believe more easy) Manner than is to be met with in any other practical Author; and the several Rules for the different Segments of a Circle and other Things pertaining to a Circle, I have illustrated by a sufficient Number of easy, useful, and entertaining Examples.

In the Second Part of the Measuring I have fully treated of the Measurement of such Solid Figures, as are bounded by Right-lined and Circular Superficies; and herein I have also observed the following Order :

First, To give such Definitions as are necessary to conceive the Form of Solid Figures.

A 4

Secondly,

Secondly, To shew the common Methods used in the Measurement of the several Sorts of Timber, with their Errours, and also how Timber may be expeditiously measured without any sensible Loss.

And Thirdly, How to find the Solidities and Superficies of various Solids, such as often occur to Workmen in the ordinary Course of their Business, which I have exemplified by various Examples suited to the several Rules and Cases.

To which I have added an Appendix, wherein is shewn the Measurement of the Conic Sections, and the Solids generated by their Rotation on their Axes; all which is sufficiently explained by Definitions, Rules, and Examples. And here the Business of measuring the Superficial and Solid Contents of Groin Arches is performed, which is made clear and easy by proper Rules and Familiar Examples; together with the Mensuration of the Superficial and Solid Contents of the Regular Solids, with proper Tables to facilitate all their Operations: And a Table of the Specific Gravity of Metals and Other Bodies with it's Uses. Also Decimal Tables of Coin, Weights, Measures, and Time, with their Construction and Uses.

The whole is illustrated with Variety of useful and pleasant Examples, so calculated and contrived, as to exercise the foregoing Propositions, and also adapted to such Uses as often occur in Life. Together with three Copper Plates, wherein are neatly engraven the various Kinds of Figures referred.

ferred to in the Definitions and Examples. I have all the Way through the several Parts of the Practical Mensuration, sought the Values of the Contents of the several Sorts of Work, and that by the most ready Methods of Aliquot Parts and Decimals; which I conceive will be of no small Use to the Workman. And I have also in their proper Places given the Allowances and Customs used amongst the Builders and Measurers.

I have omitted the Demonstrations of most of the Rules, especially those that are immediately demonstrated from Geometry or Algebra; as knowing that those Sort of Readers, for whom this Book is chiefly calculated, would not with Ease understand them, and therefore would rather serve to puzzle and amuse, than instruct them, (besides the swelling of the Book beyond it's intended Limits). And I conceive that they who would understand them, have no Occasion to look into a Book of pure Mensuration, for the learning that Science, which they cannot help being acquainted with in the studying those Parts of the Mathematics, that are requisite for the Demonstration of the Rules of Mensuration. And those who would be satisfied of the Truth of the Rules in the ensuing Treatise, may find them abstractedly demonstrated in many Mathematical Authors.

I have also omitted shewing the Use of the Sliding Rule, because I think at best it is but guessing at the Answers required; but those who would be acquainted therewith, may find it in a
Multitude

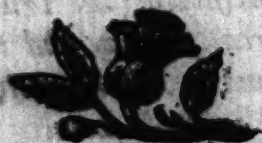
Multitude of Mathematical Books, many whereof are written for that Purpose only. Such Sliding Rules, for performing Arithmetical Operations, together, with all other Mathematical Instruments are made in the most modern, neat, and accurate Manner, by Mr T. HEATH, at the Hercules and Globe next the Fountain Tavern in the Strand.

I shall now conclude, with wishing the Reader may find as much Pleasure and Satisfaction in perusing this Treatise, as I did in compiling it; not doubting thereof, gives an additional Pleasure to his Friend and

September
27, 1738.

humble Servant,


John Robertson.





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ADVERTISEM^TENT.


ARITHMETIC, MENSURATION, GEOMETRY, ALGEBRA, TRIGONOMETRY, NAVIGATION, SURVEYING, FORTIFICATION, GUNNERY, and other useful Branches of the *Mathematics*, are taught after an easy and expeditious Manner by the *AUTHOR*; who may be spoken with, or directed to, at Mr N. ADAMS's, *Optition* to their *Royal Highnesses* the *Prince and Princesses of Wales*, near *Northumberland-House* at *Chairing-Cross*.

N. B. The *Author* also surveys Land, and delineateth Maps thereof in the most accurate Manner.



A COMPLEAT
TREATISE
OF
MENSURATION.

SECTION I.
DECIMAL FRACTIONS.
DEFINITIONS.

I.  FRACTION is some Part or Parts of an Unit ; that is, Unity or 1 is supposed to be divided into some Number of Parts, and a Fraction is one or more of those Parts.

ALL Fractions are generated by Division, thus :
When one Number is to be divided by another, and there happens a Remainder, that Remainder set over the Divisor, with a Line drawn between them (in the same manner as you would express $\frac{1}{2}$, or

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3 Farthings, viz. $\frac{1}{4}$, $\frac{1}{4}$, $\frac{1}{4}$, which are the Fractions or Parts of a Penny) constitutes a Fraction, the Value or Worth of which will always be less than an Unit: Thus, if 38 was to be divided by 6, the Quotient would be 6, and something more, viz. $6\frac{2}{3}$; for after Division is ended, there remains 2; now if this 2 be set over the Divisor 6, as above directed, it will stand thus $\frac{2}{6}$, which annexed to the Quotient 6, is $6\frac{2}{6}$, and is thus read, six Units and two parts in six of an Unit or 1. Now it is plain, that $\frac{2}{6}$ is less than an Unit, for if it were an Unit, this Unit instead of the Fraction being annexed to the 6 would make 7, which would be a wrong Quotient; for 38 divided by 6 cannot produce 7; therefore the Fraction $\frac{2}{6}$ is less than an Unit.

II. In a Fraction thus constituted, that number above the Line is called the Numerator, and that below the Line is called the Denominator.

III. A Fraction is called Vulgar, when the several Figures in the Denominator added together make a Sum greater than an Unit. Thus, in the Fraction $\frac{1}{2}$ the Denominator 2 is greater than an Unit, and is therefore a Vulgar one. And in the Fraction $\frac{1}{17}$, the two Figures in the Denominator, viz. 1 and 2 added together make 3, which is greater than Unity, and the Fraction therefore is a Vulgar one: And in the Fraction $\frac{1}{2589}$, the four Figures in the Denominator, viz. 2 and 6 and 8 and 9 added together makes 25, which is a Sum greater than an Unit or 1; and the Fraction is therefore a Vulgar one, and the same in any other.

HENCE it will be easy to conceive, that among Vulgar Fractions there may be Denominators consisting of 1, 2, 3, 4; or 10, 16, 100, &c. Places, and may all be significant Digits; which makes the Business of Vulgar Fractions very troublesome.

of MENSURATION. 3

IV. A Fraction is called a Decimal, when the Sum of the several Characters in the Denominator is equal to an Unit; or thus,

V. A Decimal Fraction, is when the Unit is supposed divided into such a number of Parts as may be expressed by an Unit, with a certain number of Cyphers annexed to it. Hence the fourth Definition is plain.

VI. A Decimal Fraction is expressed (or is always wrote) without it's Denominator, and is distinguished by prefixing a Point or Comma to the left Hand of the Figures said to denote the Fraction, the Denominator being understood always to possess a Number composed of an Unit, with as many Cyphers annexed to the right Hand as the Decimal (that is, the Numerator) hath Figures—Thus, 6 is understood to be $\frac{6}{10}$; and, 47 to be $\frac{47}{100}$; and, 763218, to be $\frac{763218}{1000000}$, &c.

HENCE ariseth the superiour Excellency of Decimal Fractions to that of Vulgar ones: For Decimals are always managed without any regard to their Denominators; whereas in Vulgar Fractions there generally ariseth more Trouble on Account of the Diversity of the Denominators, than from the Numerators themselves.

VII. A terminate Decimal, is that which ends at a certain number of Places; but an interminate no where ends.

VIII. A Repetend is a Decimal where the Figures circulate; that is, when the same Figure or Figures continually repeat, or run on the same.

If one Figure continually circulate, it is called a *single Repetend*: But if more than one Figure circulate, it is called a *compound Repetend*.

IX. THE first Place next the mark of Distinction in any Decimal Expression is called the Place

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of Primes ; and the following Places are called Seconds, Thirds, Fourths, &c.

T H E O R E M S.

I. **CYPHERS** to the right Hand of Decimals neither increase nor decrease their Value ; but Cyphers between the separating Point and the Figures of the Decimal diminish the Value ; thus ,5 is $\frac{1}{10}$, but ,05 is $\frac{1}{200}$, and ,005 is $\frac{1}{2000}$, &c. and for any other Decimal the same.

II. If the mark of Distinction in any mixed or fractional Expression be moved but one place towards the left Hand, then every Figure, and consequently the whole Expression, is but a tenth part of what it was before : But if it had been removed towards the right Hand, then the whole Expression would be ten times as much as it was before it was removed.

Note, A whole Number with a Fraction annexed thereto, is called a mixed Expression, and a Fraction without a whole Number, is called a fractional Expression.

III. **EVERY** terminate Decimal may be considered as interminate, by making Cyphers the Repetend. For (*per Theo. I.*) they do not alter the Value of the Decimal.

IV. **ANY** decimal Expression may be continued as far as you please, by repeating the circulating Figure or Figures.

Note, **ANY** Number of Decimals thus filled up, so as to have a number of Places equal to any other Decimal, are said to be made conterminous.

V. **IN** all Results, if the Repetend consist of all nines, reject them, and make the next superiour Place an Unit more ; thus, 7,23999, &c. is made 7,24, &c.

VI. **IN**

of MENSURATION. 5

VI. In all circulating Expressions, dash the first and last of the Repetend, omitting the intermediate Places; thus, $4,28\dot{3}3$, or $7,84\dot{3}4\dot{3}$ &c.

SECTION II. REDUCTION.

OR the Methods used to bring any Vulgar Fraction, or an Expression of different Denominations to it's equivalent Decimal Expressions: Or any Decimal Expression to it's Value in different Denominations.

First, To reduce a Vulgar Fraction, to it's equivalent Decimal one.

R U L E.

MAKE the Numerator a Repetend by adding thereto Cyphers, and this divide by the Denominator, and the Quotient is the Decimal sought after; which must always have as many Places as there were repeating Cyphers used—If when Division is ended there happen not so many Figures in the Quotient as there were Cyphers used to gain this Quotient, annex to the left Hand as many Cyphers as there are Places wanting, and this will be the Decimal Expression sought; minding to set before it the Comma, or mark of Distinction.

REDUCE $\frac{1}{4}$, and $\frac{1}{2}$, and $\frac{3}{4}$, and $\frac{1}{5}$, and $\frac{2}{5}$, each to their equivalent Decimals.

$4)1,00(,25$ — $2)1,0(,5$ — $4)3,00(,75$ —
 $8)5,000(,625$ — $12)7,00000(,58333, \&c.$

A N S W E R S.

$\frac{1}{4}$ is ,25. $\frac{1}{2}$ is ,5. $\frac{3}{4}$ is ,75. $\frac{1}{5}$ is ,625. and $\frac{2}{5}$ is ,88333, &c.

B 3

By

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By considering the last Example, it will be easy to conceive how Repetends are generated ; and also, to see that 3 would continually repeat.

REDUCE $\frac{1}{2}$, and $\frac{1}{3}$, and $\frac{1}{4}$, and $\frac{1}{5}$, each to their equivalent Decimal.

1st. 56) 9,0000000 (,16071428, &c.

340

400

80

240

160

480

32

2d. 286) 17,0000000 (,0594405

2700

1260

1160

1600

170

3d. 2495) 217,00 (,08697394

17400

24300

18450

9850

23650

11950

1970, &c.

4th.

of MENSURATION 7

4th. 9768) 83,000 (,0084961097

48560

93880

59680

10720

95200

72880

4504, &c.

IN the second of the foregoing Examples you may see the Generation of a compound Repetend: For after having used 7 Cyphers, I observed that if to the remainder 170 I had brought down another Cypher it would be 1700, which is the same I began with, consequently the same Figures, viz. 594405 would have repeated in the Quotient, and after them the same again, &c. Therefore I stop, and dash the first and last Figures as directed in Theorem the sixth.

IN the first, third, and fourth of the last Examples, though I have carried the Decimal on to so many Places as is there wrought, I find they do not circulate, therefore I desist carrying them any farther; and here note, that whether they circulate or not, there is seldom a Necessity of getting more than 6 Places in the Decimal or Quotient, that being sufficiently exact for most Uses.

IT may be observed that in the three last Examples, there doth not so many Figures arise, as there are Cyphers used in Division, therefore when I left off, I examined the Defect, and supply'd it with Cyphers

to

8 *A Compleat TREATISE*

to the left Hand, before which I prefixed the mark of Distinction as before directed.

Secondly. To reduce the different Denominations of Money, Weights, Measures, &c. to their equivalent Decimal Expressions.

R U L E.

PLACE the several Denominations given orderly under each other, the least being uppermost : Then reduce the least Denomination to the Decimal Part of the next Superiour ; to the right Hand of which superiour Denomination prefix the Decimal now found ; and reduce this mixed number to the Decimal of the next Superiour ; to the right Hand of which prefix the Decimal last found ; and so proceed till you arrive at the Decimal of your intended Integer.

Note. ONE Denomination is reduced to the Decimal of the next Superiour by dividing the lesser Denomination by the number thereof contained in one of the next Superiour.

Reduce 10 s. 8 d. — 13 s. 10½ d. — 15 s. 9½ d. and 19 s. 11½ d. each to it's equivalent Decimal of a Pound Sterling.

| | | | |
|-----------|-------------|--------------|--------------|
| 12 8 | 4 2 | 4 3 | 4 1 |
| 20 10,6 | 12 10,5 | 12 9,75 | 12 11,25 |
| 0,53 | 20 13,875 | 20 15,8125 | 20 19,9375 |
| | 0,69375 | 0,790625 | 0,996875 |

A N S W E R S

10 s. 8 d. is, 53 l. — 13 s. 10½ d. is, 69375 l. — 15 s. 9½ d. is, 790625 l. — and 19 s. 11½ d. is, 996875 l.

REDUCE

of MENSURATION. 9

REDUCE 10 oz. 18 dwts. 16 grs. and 3 grs. 16 lb. 12 oz. each to their equivalent Decimals.

| | | | | | |
|----|----------|----------|----|----------|---------------|
| 24 | { 4 6 | 16 (4 | 16 | { 4 4 | 12 (3 |
| 20 | | 18,8 | 28 | { 4 7 | 16,75 |
| 12 | | 10,98 | 4 | | (41875 |
| | | 0,98 | | | 3,598214, &c. |
| | | | | | 0,899553, &c. |

A N S W E R S.

10 oz. 18 dwts. 16 grs. is ,98 of a lb Troy, and 3 grs. 16 lb 12 oz. is ,899553 of a C. wt. Averdupoise.

REDUCE 10 Inch. 09 Sec. 07 Parts, a Duodecimal; and 48 Min. 37 Sec. 54 Thirds, a Sexagesimal; each to their equivalent Decimal.

| | | | |
|----|---------------|----|----------|
| 12 | 7 | 60 | 54 |
| 12 | 9,588 | 60 | 37,9 |
| 12 | 10,79868 | 60 | 48,6318 |
| | 0,899884, &c. | | 0,810527 |

A N S W E R S.

10 Inch. 09 Sec. 07 Parts, is ,899884, &c. of a Foot, and 48' 37" 54''' is ,810527 of a Degree.

AFTER the same Manner may any other sorts of contract Numbers be reduced.

BUT the Decimal Parts of a Pound Sterling may be expressed in one Line; thus: Write half the Number of Shillings in the Place of Primes, and reduce the Pence and Farthings into Farthings, which must possess the Places of Seconds and Thirds; and if the Shillings be odd, increase the Place of Seconds with 5: But increase the Place of Thirds with as many Units as there are times 24 in the Pence and Farthings.

LASTLY,

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LASTLY, If half the Number of Farthings in the Pence and Farthings [rejecting Sixpences] be divided by 12, the Quotient written after the three Places already found, shall compleat the Decimal.

REDUCE 10 s. 8 d.—13 s. 10½ d.—15 s. 09½ d.—19 s. 11½ d.—1 s. 10½ d.—8½ d.—2½ d. and ¼ d. each to their equivalent Decimal of a Pound Sterling.

| | | | |
|----------------|------------------|-------------------|-------------------|
| 10 s. 8 d. | 13 s. 10½ d. | 15 s. 09½ d. | 19 s. 11½ d. |
| <u>l. ,583</u> | <u>l. ,69375</u> | <u>l. ,790625</u> | <u>l. ,996875</u> |

| | | | |
|--------------------|--------------------|-------------------|-------------------|
| 1 s. 10½ d. | 8½ d. | 2½ d. | ¼ d. |
| <u>l. ,0927083</u> | <u>l. ,0364583</u> | <u>l. ,010416</u> | <u>l. ,003125</u> |

By explaining one of these Examples, all the rest will be easy.

IN the fifth, viz, 1 s. 10½ d. First half of one is 0, write 0 in the Place of Primes; then 4 times 10 d. is 40 Farthings, and ½ is 41, and because here is one 24, I add 1 to 41 and it makes 42; and because there is one Shilling, I increase the second Place, viz. 4 with 5, which now makes 92, this written after the 0, makes ,092. Lastly in 41 Farthings, rejecting 24 or Sixpence, there is 17 remains, half of which 8,5, which divided by 12, gives 7083 to be added to ,092. If this Process be well compared with the last Rule, the whole will be very plain.

THIRDLY, To find the Value of a Decimal.

R U L E.

MULTIPLY the given Decimal by the Number of the next lesser Denomination, contained in one

of MENSURATION. 11

one of that Denomination which the Decimal respects as it's Integer; and from the Product cut off as many Places to the right Hand, as there are in the given Decimal; and this Decimal thus cut off, multiply by the next lesser Denomination, and from this Product cut off as before, and thus proceed till you are arrived at the least Denomination: Lastly, the several Denominations cut off on the left Hand are the Answers.

WHAT is the Value of ,72896 l.—,92384 l. and ,089235 l.

| | | |
|-----------------|-----------------|-----------------|
| ,72896 | ,92384 | ,089235 |
| 20 | 20 | 20 |
| <u>14,57920</u> | <u>18,47680</u> | <u>1,784700</u> |
| 12 | 12 | 12 |
| <u>6,95040</u> | <u>5,72160</u> | <u>9,416400</u> |
| 4 | 4 | 4 |
| <u>3,80160</u> | <u>2,88640</u> | <u>1,665600</u> |

ANSWERS. ,72896 l. is 14 s. 6½ d.—,92384 l. is 18 s. 5½ d. and ,089235 l. is 1 s. 9½.

THIS is so plain it needs no Explanation.

WHAT is the Value of ,87628 of a lb Troy, and of ,798645 of a G.wt. Avoirdupoize?

| | |
|------------------|-----------------|
| ,87628 | ,798645 |
| 12 Ounces. | 4 Quarters. |
| <u>10,51536</u> | <u>3,194580</u> |
| 20 Penny weight. | 28 Pound. |
| <u>10,30720</u> | <u>1556640</u> |
| 24 Grains. | <u>389160</u> |
| <u>122880</u> | <u>5,448240</u> |
| <u>61440</u> | 16 Ounces. |
| <u>7,37280</u> | <u>7,171840</u> |

ANSWERS

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A N S W E R S.

,87628 lb is 10 oz. 10 dwts. 7 grs. and
,798645 C. wt. is 3 grs. 5 lb 7 oz.

BUT the Value of the Decimal of any part of
a Pound Sterling may be expressed it one Line
thus.

DOUBLE the Place of Primes for Shillings, and
if the second Place be 5, or exceed 5, increase the
double of the Primes by one for Shillings: Then
the Figures in the second and third Places [rejecting
5 in the second Place] are so many Farthings,
abating one for every 24, and the Remainder divided
by 4 gives the Pence and Farthings.

WHAT are the Values of ,92763 l. and ,87638 l.
and ,09937 l. and ,0428 l. and ,0095 l.

,92763 l. ,87638 l. ,09937 l. ,0428 l.
18 s. 6½ d. 17 s. 6½ d. 1 s. 11½ d. 10½ d.

,0095 l.
2½ d.

S E C T I O N III.

ADDITION and SUBTRACTION.

R U L E.

ADD or subtract as in whole Numbers, and
from the Sum or Difference cut off as many De-
cimal Places as are the greatest Number of Deci-
mals

of MENSURATION. 13

imals in any of the given Expressions : But observe that the separating Commas in each Expression be placed directly underneath each other ; for then Units, &c. (if any) will fall under Units, &c. and Primes, Seconds, Thirds, &c. under Primes, Seconds, Thirds, &c. for so they ought to do.

EXAMPLES in ADDITION.

| | | |
|------------------|--------------------|---------------------|
| 347,256 | 3468,04973 | 8267,4031 |
| 5,61739 | 24,3675 | 8,965 |
| ,1725 | 148,952 | 453,720853 |
| 43,. | 37,284695 | 76,98124 |
| 36,5 | 5,125 | ,2738107 |
| <u>432,54589</u> | <u>3683,778925</u> | <u>8807,3440037</u> |

EXAMPLES in SUBTRACTION.

| | |
|----------------------------------|------------------|
| From 384,76215 Minuend. | From 426,8 |
| Take 86,2095. Subtrahend. | Take 379,604832 |
| <u>298,55265</u> Differ. or Rem. | <u>45,195168</u> |

If Interminates are to be added, that have single Repetends.

R U L E.

MAKE all the Repetends conterminous, that is, continue the repeating Figure till the Decimal Places in each are equal ; observing that the repeating Figures be carried one Place farther to the right Hand than the greatest Number of Decimal Places in any of the terminate Expressions : Then add as before ; only increase the Sum of the right hand Row with as many Units as it contains Nines, and the Figure in the Sum under that Place will be a Repetend.

C

EXAM-

14 *A Compleat TREATISE*

EXAMPLES.

| | | | |
|-----------|----------|----------|-----------|
| 8439,6548 | 5391,357 | 217,8498 | 876,298 |
| 281,048 | 72,38 | 42,178 | 5,8764289 |
| 7042,38 | 187,21 | ,528 | ,03588 |
| 9,837 | 4,2968 | 58,30048 | 628,45938 |

IN each of these Examples there are Repetends, and before they can be added, they must be made conterminous, and then they will stand as follows:

| | | |
|--------------------|------------------|-------------------|
| 8439,6548 | 5391,357 | 217,849866 |
| 281,04866 | 72,3888 | 42,178666 |
| 7042,38555 | 187,2111 | ,528333 |
| 9,83777 | 4,2968 | 58,30048 |
| <u>15772,89488</u> | <u>5055,2538</u> | <u>318,850146</u> |

876,29833333
 5,8764289
 ,03588666
 628,45938888
1510,66501778

HERE you may observe that in each Example the Repetends are carry'd one Place farther than the terminate Expressions, and to the Sum of that Row, there are as many Units added, as there were Nines in the Sum.

IF Subtraction is to be performed with Interminates that have single Repetends.

R U L E.

MAKE all conterminous as in Addition: But if the right hand Figure of the Subtrahend (being a Repetend) be bigger than the Figure over it in the Minuend

of MENSURATION. 15

Minuend, instead of borrowing 10 as in Subtraction of whole Numbers or Terminates ; borrow only 9 and this only in the right hand Figure, and this Figure of the Remainder will be a Repetend.

| | | | |
|---|------------------------------------|-------------------------------------|-----------------------------------|
| From 476,3 ¹ / ₂ | 289,576 | 325,791 ¹ / ₂ | 643,9207 |
| Take 84,769 ¹ / ₂ | 92,584 ¹ / ₂ | 37,095 | 583,7 ¹ / ₂ |
| <hr/> | <hr/> | <hr/> | <hr/> |

THESE Examples being made conterminous as before directed, will stand thus :

| | | | |
|--------------------------------------|-------------------------------------|-------------------------------------|---------------------------------------|
| 476,3 ¹ / ₂ 22 | 289,576 ¹ / ₂ | 325,791 ¹ / ₂ | 643,9207 ¹ / ₂ |
| 84,769 ¹ / ₂ | 92,584 ¹ / ₂ | 37,095 ¹ / ₂ | 583,7 ¹ / ₂ 666 |
| <hr/> | <hr/> | <hr/> | <hr/> |
| 391,5524 | 196,991 ¹ / ₂ | 288,696 ¹ / ₂ | 60,1540 ¹ / ₂ |
| <hr/> | <hr/> | <hr/> | <hr/> |

SECTION IV.

MULTIPLICATION.

R U L E.

PLACE and multiply as in whole Numbers, and from the Product towards the right Hand, cut off as many Places for Fractions, as there are fractional Parts in both Expressions or Factors together. But if it so happen, that there are not so many Places in the Product, supply the Defect with Cyphers to the left Hand.

C 2

3684,7928

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| | | |
|----------------------|----------------|---------------|
| 3684,7928 | ,2365 | ,0347 |
| 84,216 | ,2435 | ,0236 |
| <hr/> 221087568 | <hr/> 11825 | <hr/> 2082 |
| 36847928 | 7095 | 1041 |
| 73695856 | 9460 | 694 |
| 147391712 | 4730 | <hr/> 0081892 |
| 294783424 | <hr/> 05758775 | |
| <hr/> 310318,5104448 | | |

IN the first Example there being 4 Decimal Places in the Multiplicand, and 3 in the Multiplier, which together are 7 ; I therefore cut off 7 Figures from the right Hand of the Product for Decimals ; those to the left Hand being Integers.

IN each of the second and third Examples the Decimal Places in both Factors are 8, but there turns out in the second Example only 7 Figures in the Product, and but 5 in the third Example's Product : Therefore in the second Example I annex 1 Cypher, and in the third 3 Cyphers, to supply the Defect.

IN multiplying with single Repetends there are three Cases.

CASE I.

IF the right hand Figure of the Multiplicand be a Repetend.

RULE.

MULTIPLY the multiplicand as before, by every Figure in the Multiplier ; observing to increase the right hand Figure of each resulting Line, by as many Units as there are Nines in the first Product of that Line ; and the right hand Figure of each Line will be a Repetend ; therefore in the adding the several Lines together, they must be made conterminous, as taught in Addition.

EXAM-

of MENSURATION. 17

EXAMPLES.

First. $821,734\frac{1}{2}$
 $\frac{8}{10}$
657,38764

Second. $3796,273$
 $\frac{4,6}{10}$
22777640
151850983
1746,28578

Third. $8946,83706$
 $\frac{48,57}{100}$
6262785940
44734185333
715746965333
3578734820666
434547,8763280

CASE II.

WHEN the Multiplier is a single Repetend.

RULE.

MULTIPLY by it as tho' it were a terminate Digit, setting the Product one Place forwarder than ordinary, towards the left Hand, and divide the Result by 9, continuing the Quotient (if needful) till you arrive at a Repetend; then beginning at the Place under the right hand Figure of the Multiplicand, cut off for Fractions as before, and you'll have the true Product.

C 3

EXAM-

18 A Compleat TREATISE

EXAMPLES.

| | | | |
|---------------|--------------------|----------------|----------------------|
| <i>First.</i> | 438,6297 | <i>Second.</i> | 5820,39462 |
| | <u>9) 26317782</u> | | <u>9) 4074276234</u> |
| | 292,41988 | | 4526973598 |
| | | | 1746118386 |
| | | | <u>2198,8157458</u> |

| | | | |
|---------------|------------------|--|------------------|
| <i>Third.</i> | 47,63 | | 2,848 |
| | <u>2,848</u> | | <u>47,63</u> |
| | 9) 23815 | | 8538 |
| | 26468 | | 170783 |
| | 19052 | | 199188 |
| | 38104 | | 1138222 |
| | 9526 | | <u>135,53388</u> |
| | <u>135,53388</u> | | |

THE third Example I have wrought by both Cases, where you may observe that they prove each other; the Result turning out exactly the same.

In this second Case, if there are any other Figures in the Multiplier, beside the Repetend, multiply by them like terminate Digits.

CASE III.

WHEN the Multiplicand and Multiplier are each a single Repetend.

RULE.

In multiplying the Multiplicand by each Figure in the Multiplier, observe the Direction given in CASE I. but the first Line (or that Line produced by multiplying the Multiplicand by the Repetend in the

of MENSURATION. 19

the Multiplier) must be managed as directed in
CASE II.

EXAMPLES.

| | |
|--|--|
| $ \begin{array}{r} 463,9704 \\ 8,647 \\ \hline 9) 927940 \\ \hline 1031048432, & c. \\ 18558817 \\ 278382288 \\ 3711763855 \\ \hline 4009,7356843, & c. \end{array} $ | $ \begin{array}{r} 862357,94 \\ 35,16 \\ \hline 9) 517414758 \\ \hline 57490528148 \\ 86235794 \\ 4311789641 \\ 25870737866 \\ \hline 30326253,59814 \end{array} $ |
| $ \begin{array}{r} 5392768 \\ 953,7 \\ \hline 9) 37749358 \\ \hline 41943732098, & c. \\ 16178298 \\ 269638277 \\ 48534898 \\ \hline 514,350094765432098 \end{array} $ | |

By dividing by 9. in each of these Examples there results a compound Repetend, now having finished the Multiplication, I make all the Lines conterminous, but no farther than the first Figure of the compound Repetend, then having added the several Lines together as before directed, I dash the right hand Figure of the Product, for the first Figure of a compound Repetend; the rest of which I set orderly after as they follow in the given compound Repetend.

WHEN there happens many Decimal Places in each Factor, there doth usually arise many useless Figures

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Figures in the Product, to avoid which observe the following

R U L E.

HAVING determined how many Decimal Places are to be had in the Product, under THAT Place in the Multiplicand set the Unit's Place of the Multiplier, and invert (or turn round) the Order of all the other Places of it: Then in multiplying, begin at that Figure of the Multiplicand which stands over the Figure you are then multiplying with; setting down the first Figure of every Product directly underneath one another, observing to have a due regard in carrying the Increase which would arise out of the Figures to the right Hand of that Figure in the Multiplicand you then begin with, and in the Product the lowest Place is uncertain.

BUT if from the Figures omitted you carry'd 1 from 5 to 15; 2 from 15 to 25; 3 from 25 to 35; &c. instead of carrying 1 for every 10, &c. the right hand Place in the Product would be generally exact.

E X A M P L E S.

MULTIPLY 384,672158 by 36,8345
Now seeing there would be 10 Decimal Places in the Product, whereof the greatest Part are unnecessary; therefore I intend to keep only 4 Decimal Places in the Product.

of MENSURATION. 21

| | | |
|----------------------|----------------------|---------------------|
| 384,672158 | Multiplicand. | 384,672158 |
| 5438,63 | Multiplier inverted. | 36,8345 |
| 115401647 . | | 1923 360790 |
| 23080329 . . | | 15386 88632 |
| 3077377 . . . | | 115401 6474 |
| 115402 | | 3077377 264 |
| 15387 | | 23080329 48 |
| 1923 | | 115401647 4 |
| 14169,2065 | | 14169,2066 038510 |

HERE I have wrought the Example both ways, by which may be easily seen what is saved by the contracted way.

IN this Example because I intended to keep 4 Decimal Places in the Product, I set 6, the Unit's Place of the Multiplier under 1, the 4th Place in Decimals of the Multiplicand, and invert the Order of all the rest of the Figures: Then I say 3 times 8 is 24, and carry 2; 3 times 5 is 15, and 2 is 17, now set down the 7 and carry 1, &c. because this is the Product arising by Multiplying the 5 that stands over the 3.

AGAIN, 6 times 8 is 48, and carry 4; 6 times 5 is 30, and 4 is 34, and carry 3; 6 times 1 is 6, and 3 is 9. Now being come to the Figure over the 6, I set down 9, &c.

AGAIN, 8 times 5 is 40, and carry 4; 8 times 1 is 8, and 4 is 12, and carry 1; 8 times 2 is 16, and 1 is 17; now being come to the Figure over the 8, I set down 7, and carry 1, &c. Proceeding in like Manner with every Figure in the inverted Multiplier, till all is done.

Mul.

22 *A Compleat TREATISE*

MULTIPLY 3,141592 by 52,7438, and to reserve 4 Decimal Places in the Product.

$$\begin{array}{r}
 3,141592 \\
 \times 52,7438 \\
 \hline
 8347,25 \\
 1570796 \\
 62831 \\
 21991 \\
 1257 \\
 94 \\
 25 \\
 \hline
 165,6994
 \end{array}$$

MULTIPLY 257,356 by 76,48, and to have the Product only in whole Numbers.

| | |
|---|--|
| $ \begin{array}{r} 257,356 \\ \times 76,48 \\ \hline 18015 \\ 1544 \\ 103 \\ 20 \\ \hline 19682 \end{array} $ | $ \begin{array}{r} 257,356 \\ \times 7648 \\ \hline 2058848 \\ 1029424 \\ 1544136 \\ 1801492 \\ \hline 19682,58688 \end{array} $ |
|---|--|

SECTION V.

DIVISION.

RULE.

DIVIDE as tho' all were Integers ; but so place the separating Comma in the Quotient, that the Sum of the decimal Places in the Divisor and

of MENSURATION. 23

and Quotient may be equal to those in the Dividend——But if the Decimal Places in the Divisor be more than those in the Dividend add Cyphers as Decimals to the Dividend, till the Number of Decimal Places in the Dividend is at least equal to those in the Divisor.

BUT if there arise not so many Places in the Quotient as the Rule requires, supply the Defect with Cyphers to the left Hand.

$$43,6) 3424,6056 (78,546$$

$$\underline{3726}$$

$$\underline{2380}$$

$$\underline{2005}$$

$$\underline{2616}$$

$$,675) 3877875,000 (5745000$$

$$\underline{5028}$$

$$\underline{3037}$$

$$\underline{3375}$$

$$\underline{000}$$

$$,347) ,0081892 (,0236$$

$$\underline{1249}$$

$$\underline{2082}$$

If these Examples be compared with the foregoing Rule, it will be very easy to see how they are performed, and the Quotient rightly adjusted.

If the Dividend be a Repetend, bring down the repeating Figure to the Remainders in order to have your Quotient to it's intended Exactness.

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472) 833 (176553, &c.

3613

3093

2613

2533

1733

317, &c.

54283) 876527 (016147, &c.

333697

79997

257147

400157

20176, &c.

If the Divisor be a Repetend—Then imagine the separating Point of the Divisor moved as many Places to the left Hand as the Divisor's Repetend consists of Places, and then subtract it from the Divisor itself: Also remove the separating Point of the Dividend towards the left Hand as many Places as the Divisor's was; and this subtract from the Dividend itself: Then divide with those Remainders as in the foregoing Directions, and the Result is the Quotient sought after.

EXAMPLES

of MENSURATION. 25

EXAMPLES.

First. $8946,83708) 434547,876328 (48,57$
 $\underline{894,68370} \quad \underline{43454,7876328}$
 $8052,15336) 391093,0886952$
 $\underline{6900695429}$
 $\underline{4589727415}$
 $\underline{5636507352}$
 $\underline{0000000000}$

It will be easy to see how this Example is performed, by observing the Rule; and by comparing it with Example 3d to Case I. of Repetends in Multiplication, you will see how these two Rules prove each other.

Second. $748,64) 47,464057 (,0634$
 $\underline{74,86} \quad \underline{4,746405}$
 $673,78) 42,717652$
 $\underline{229085}$
 $\underline{269512}$
 $\underline{0}$

Third. $2,84) 135,53387 (47,63$
 $\underline{284} \quad \underline{13,55338}$
 $2,561) 121,98043$
 $\underline{19540}$
 $\underline{16134}$
 $\underline{7683}$
 $\underline{0}$

D

Fourth.

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Fourth. 76,44) 82937643 (108,44

764 8293764

68,83) 74643879

58138

9747

32159

4627

DIVISION like Multiplication may be contracted as follows :

HAVING determined of what Value the first Figure in the Quotient will be : Let each Remainder be a new Dividend, and for each such new Dividend prick off one Figure from the right Hand of the Divisor : Observing at each Multiplication to have regard to the Increase of the Figures so cut off ; and the last Figure in the Quotient is generally uncertain.

EXAMPLES.

384,672158) 14169,2666239510 (36,8345

..... 1154016474

262904188 .

230803294 .

32100894 .

3077372 .

1227122 .

1154016 .

173106 .

153868 .

19238 .

19233 .

9,365407

of MENSURATION. 27

9.365407) 87.076326 (9297655

84 288663

2787663

1873081

914582

842886

71696

65557

6139

5619

520

468

52

46

6

THIS will not be difficult if it be carefully examined.

SECTION VI.

Of COMPARISON or PROPORTION.

DEFINITIONS.

THE comparing things of a like Kind to one another may be considered two Ways:

First, By how much one thing exceeds, or is greater than another, and this is called *Difference*.

Secondly, WHAT Part or Parts one thing is of another, and this is called *Ratio*. And two or more Ratio's make a Proportion, viz.

D 2

WHEN

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WHEN one Number is to be divided by another ; then the dividing Number is said to be in the same Proportion to the Number to be divided ; as Unity or one is in proportion to the Quotient ; that is,

THE dividing Number is as often contained in the Number to be divided : As Unity or 1 is contained in the Quotient. Therefore Ratio (and two Ratios make Proportion) is nothing more than how often one Number is contained in another ; that is, it is the Quotient of one Number divided by another.

II. THERE cannot be less than two Numbers or Terms in any Proportion : The first of which or the Term by which the Comparison is made is called the *Antecedent* : And the Second ; or the Term to which the first is compared, is called the *Consequent*.

III. WHEN two Differences are equal, the Numbers or Terms by which these Differences were made, are said to be in Arithmetical Proportion or rather Progression. Thus 4, 6, 8, 10, are Numbers in Arithmetical Proportion, because the Difference between every one of the Terms are equal and the like in other Numbers.

BUT when two Ratios (*i. e.* Quotients) are equal, the Numbers or Terms by which these Ratios were made, are said to be in Geometrical Proportion. Thus 3, 12, 4, 16, are Numbers in Geometrical Proportion, because twelve divided by 3, gives 4 ; and 16 divided by 4 gives 4 also ; that is, they give equal Quotients ; therefore the Numbers are in Geometric Proportion ; and the same is to be understood of other Numbers that will give equal Quotients.

IV. WHEN

of MENSURATION. 29

IV. WHEN of several Terms or Numbers, the Quotient of the first and second is the same with that of the second and third, and the same with the third and fourth, &c. those Numbers are said to be in continued Geometric Proportion: Thus, 3, 12, 36, 108, &c. are Numbers in continued Geometric Proportion, for the Quotient of any two adjacent Terms is 3.

THEOREMS.

I. IF there are three Terms in Geometrical Proportion, the first Term multiplied by the third; gives a Product equal to the second Term multiplied by itself; that is, the second Term squared.

HENCE the second Term is called *a mean Proportional between the Extreams*: viz. the first and third Terms.

II. IF there are four Terms in Geometrical Proportion; the Product of the two extream Terms is equal to the Product of the two mean Terms.

HENCE the second and third Terms are called *mean Proportionals between the first and fourth Terms*.

NOW it is evident, that if the Product of the second and third Terms be divided by the first Term, the Quotient will be the fourth Term.

AND hence ariseth the Method of operating the Rule of three.

THE Rule of three is so called, because three Numbers or Terms are given to find a fourth: Thus, if the Numbers 3, 12, and 18 were given to find a fourth Proportional.

NOW multiplying the second Term by the third, or (which is all one) the third by the second, viz. 18 by 12, the Product will be 216; this 216 divided by the first Term 3, gives 72 in the Quotient, for the fourth Term.

D 3

Now

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Now by comparing of the Terms together, you will find that the second Term 12 as often contains the first Term 3; as the fourth Term 72, contains the third Term 18: Also that the third Term 18 as often contains the first Term 3, as the fourth Term 72 contains the second Term 12; that is, the Ratio or Proportion of 3 to 12 is the same as the Ratio of 18 to 72. Also the Ratio of 3 to 18 is the same as the Ratio of 12 to 72; and the like in any other Numbers. And this is the Doctrine of Proportions;

THE Rule of three being mostly concerned in finding a fourth Number proportional to three Numbers or Terms given, differing in Signification and Denomination, the greatest Difficulty lies in stating the Terms; that is, in placing them in proper Order to be multiplied and divided according to the foregoing Directions; to do which observe the following Directions.

CONSIDER what Sort or Quality the Term required is of: And of the three Terms given, set that which is of the same Quality with the Term required, for the second Term; and the remaining two Terms for Distinction sake, call the *Antecedential Terms*.

THEN by the Tenour of the Question, you'll easily be able to judge whether the fourth Term, or Term required, will be less or greater than the second Term.

HAVING considered that, place the Antecedential Terms for first and third in the same Order; that is, if the second term be less than what the fourth will be, place the least of the Antecedential Terms for the first Term, and the other for the third: But if the second Term is greater than what the fourth will be; place the greater of the Antecedential

of MENSURATION. 31

essential Terms for the first Term, and the other for the third : And then the three Terms are stated fit for Solution,

THEN if any, or all of the Terms be of different Denominations, reduce the lesser Denominations to the decimal Part or Parts of the greater Denomination ; as taught in SECT. II. observing that the lesser Denominations in the first and third Terms be reduced to the decimal Part or Parts of one and the same Denomination.

THEN having multiplied the second and third Terms, together by the Rules in SECT. IV. and divided the Product by the first Term, and adjusted the Quotient by the Rules given in SECT. V, you may then call the Quotient the *fourth Term*, or the *Thing sought after* ; and may proceed to value it by the Rules given in SECT. II.

QUESTION I

WHAT will $326\frac{1}{4}$ lb of Tobacco come to, at 3 s. 6 d. for $1\frac{1}{2}$ lb ?

HERE the Quality of the fourth Term is Money, viz. the worth of $326\frac{1}{4}$ lb of Tobacco : And among the three Terms given, one is Money, viz. 3 s. 6 d. the worth of $1\frac{1}{2}$ lb.

Now I can very easily see that $326\frac{1}{4}$ lb will come to more than $1\frac{1}{2}$ lb, therefore the fourth Term will be greater than the second ; and the Terms stated will stand thus : If $1\frac{1}{2}$ lb cost 3 s. 6 d. what will $326\frac{1}{4}$ lb come to ; and the Terms reduced into Decimals will stand thus :

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If 1,5 lb ———, 175 l. ——— 326,25 lb

75

163125

554625

1,5) 57,09375 (38,0625 L

120

93

37

75

HERE the third Term being multiplied by the second; and the Product being divided by the first Term gives 38,0625 *l.* in the Quotient; which being valued as directed in SECT. II. gives 38 *l.* 1 *s.* 3 *d.* the Money that 326½ *lb.* of Tobacco will come to, at the Rate of 3 *s.* 6 *d.* for 1½ *lb.*

QUESTION II.

WHAT is the worth of 19 oz. 3 dwts. 5 grs. of Gold, at 2 l. 19 s. 4 d. per Ounce?

If 1 oz. — 2 l. 19 s. 4 d. — 19 oz. 3 dwts. 5 grs.

| | | | |
|----|---|---|----------|
| 24 | 5 | 4 | 5 |
| | 2 | 6 | (125 |
| 20 | | | 3,2087 |
| | | | 0,160416 |

Or thus in Decimals :

1 lb oz. — 2,96 l. — 19,160416 oz.

2,96

9) 114962500

127736x11

P724437500

3832083833

56,842,5694

BECAUSE

of MENSURATION. 33

BECAUSE the first Term here is Unity or 1, which neither multiplies nor divides, therefore the Answer is produced by multiplying the second and third Terms together; and the Product being valued as taught in SECT. II. gives 56 *l.* 16 *s.* 10½ *d.* the Value of the Gold, as sought after.

QUESTION III.

WHAT is the Worth of 827½ Yards of Painting at 10½ *d.* per Yard.

IF 1 *yd.* — ,04375 *l.* — 827,750 0 *yd.*

57 340,0

33 1100

2 483 2

579 45 Ans. 36 *l.*

41 42 4 *s.* 3½ *d.*

36,214 0

Now because there would be 7 decimal Places in the Answer, whereof 4 are more than sufficient, I therefore in order to get 4 Decimals in the Product, set the Place of 0 Units under the fourth Decimal in the Multiplicand, and invert the Order of the rest as directed in SECT. IV.

QUESTION IV.

LENT my Friend 34 *l.* for ¼ of a Year, how much ought he to lend me ⅓ of a Year, to requite my Kindness?

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75 y. — 34 l. — 625 y.
 34
 2500
 1875
 75) 21,25 (28,833 } ANSWER.
 625
 250
 25

QUESTION V.

If $\frac{1}{4}$ of a Yard of Cloth, that is $2\frac{1}{4}$ Yards broad, make a Garment, how much of another Sort, that is but $\frac{1}{4}$ of a Yard wide, will make the same Garment?

8 y. b. — 75 y. l. — 2,25 y. b.

75
 1125
 1575
 8) 1,6875 (2,1093 75

Answer. 2 Yards and 1 Nail.

QUESTION VI.

If when the Bushel of Wheat cost 4 s. 9 d. the Penny Loaf weigh'd 10 $\frac{1}{2}$ Ounces, what will it weigh when the Bushel of Wheat is sold for 8 s. 10 d.

of MENSURATION. 35

| | | | |
|------|---------|----------|---------------------|
| d. | oz. | d. | |
| 4416 | 10,6 | 2375 | |
| | | 10,6 | |
| | | 9) 14250 | 7. |
| | | 15833 | 12 |
| | | 23750 | 8,4 |
| 4416 | 2,58333 | 5,7 | |
| 416 | 25333 | | |
| 4000 | 2,28000 | | |
| | | | ANSWER, |
| | | | 5 $\frac{2}{3}$ oz. |

FOR the speedy operating those Examples in the Rule of Three, where the first Term is Unity or 1; there are several compendious Rules given in the Books of Arithmetic, and by the Authors called *Practice* (which is nothing more than the Application of the Doctrine of Aliquot Parts) and these Rules are nearly as many as there have been Writers of Arithmetic; Therefore, as it would be an endless Task, to give all the easy Methods of Operation adapted to particular Cases; I shall only mention some general Directions, whereby with the Judgment of the Operator, the most common Cases may be expeditiously solved. And,

First. AN Aliquot Part is such a Part of some whole, that the Aliquot Part taken a certain Number of Times will make the whole: Thus three is an Aliquot Part of 18, because 3 taken 6 times will make 18.

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THE following Tables shews the Aliquot Parts of a Pound, and of a Shilling.

| | | | | | |
|-----------------------------|-------------|------------------------------------|----------------|--------------------|-------------|
| 10 s. is $\frac{1}{2}$ | of a Pound. | 6 d. is $\frac{1}{2}$ | of a Shilling. | and $\frac{1}{20}$ | of a Pound. |
| 6 s. 8 d. - $\frac{1}{3}$ | | 4 d. - $\frac{1}{3}$ | | and $\frac{1}{20}$ | |
| 5 s. - - - - $\frac{1}{4}$ | | 3 d. - $\frac{1}{4}$ | | and $\frac{1}{20}$ | |
| 4 s. - - - - $\frac{1}{5}$ | | 2 d. - $\frac{1}{5}$ | | and $\frac{1}{20}$ | |
| 3 s. 4 d. - $\frac{1}{6}$ | | 1 $\frac{1}{2}$ d. - $\frac{1}{6}$ | | | |
| 2 s. 6 d. - $\frac{1}{8}$ | | $\frac{3}{4}$ d. - $\frac{1}{8}$ | | | |
| 2 s. - - - - $\frac{1}{10}$ | | | | | |

IN all Cases where the Value of some given Things are required.

TAKE the most convenient Aliquot Parts of the greater Denomination of the Things given to be valued, for the several inferior Denominations of the Price: And for the Value of the inferior Denominations of the Things given to be valued, take the most convenient Aliquot Parts of the Price.

SECT.

ART

SECTION VII.

EXTRACTION of the SQUARE
ROOT.

EXTRACTION of the square Root is the finding such a Number out of any Number given, as being multiplied into itself will either produce the given Number, or come very near it.

THEREFORE a square Number is that which is produced by the Multiplication of two equal Numbers together: Thus 9 is a square Number, as being produced by multiplying 3 by 3. Also 144 is a square Number, equal to 12 times 12.

THE following Table exhibits the Roots and Squares of all single Numbers.

ROOTS: 1, 2, 3, 4, 5, 6, 7, 8, 9.

SQUARES. 1, 4, 9, 16, 25, 36, 49, 64, 81.

WHEN any Number is given to extract the square Root out of it, the first thing must be to point it into Periods of two Figures each; that is, always beginning at the left hand Figure, or Figure that stands in the Place of Units, over which put a Point, and point every second Figure counting from the right Hand towards the
E Left;

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Left ; but in pointing of Decimals, point every second Figure from the left Hand towards the Right.

THEN seek the greatest Square contained in the first Period towards the left Hand, whose Root place in the Quotient as in Division, and the Square of the Root set under the Period out of which it was taken ; from which Period subtract it, and to the Remainder bring down the next Period, which call a *Resolvend*.

THEN double the Root or quotient Figure for a Divisor, and seek how often it may be had in all the Figures of the Resolvend, excepting the right hand Figure and the Figure resulting, place both in the Quotient, and on the right Hand of the Divisor ; then multiply the increased Divisor by the Figure last put in the Quotient, and the Product place under the new Resolvend, (observing to put Units under Units from which subtract it, and to the Remainder bring down the next Period for a new Resolvend, and double all the Quotient for a Divisor ; and proceed as before till all the Periods are brought down.

If at last there happen to be a great Remainder, and you are willing to have the Root more accurate by increasing it with a decimal Fraction. Then to the Remainder annex two Cyphers, and prosecute the Work as before, &c. always adding two Cyphers to the Remainder, till you arrive at your desired Exactness.

EXAMPLE I.

WHAT is the square Root of 132496.

13249

of MENSURATION. 39

$$\begin{array}{r}
 132496 \text{ (364)} \\
 9 \\
 \hline
 66 \overline{) 424} \\
 \underline{396} \\
 724 \overline{) 2896} \\
 \underline{2896} \\
 \hline
 \end{array}$$

THE first Period towards the left Hand is 13, the greatest Square in which is 9, whose Root 3, I place in the Quote, and the Square 9 I place under the 13, and subtracting, there remains 4, to which I bring down the next Point 24, which makes the Resolvend 424; to the left Hand of which I draw a curv'd line, and at some Distance therefrom I put the double of 3, viz. 6, and inquire how oft this 6 may be had in 24, and I find 6 times, which 6 I place in the Quote, and also on the right Hand of the Divisor 6, and the increased Divisor is 66, which I multiply by the 6 in the Quote, and it gives 396, which subtracted from 424 leaves 28, to which the next Period 96 is brought down; and 36, the Quotient, doubled makes 72, the Divisor, which in 289 goes 4 times, which 4 I place in the Root and Divisor, and the new Divisor 724 multiplied by 4, gives 2896, so that nothing remains: Therefore I conclude 364 to be the true Root; for 364 multiplied by 364 gives 132496 the first Number given.

EXAMPLE II.

WHAT is the Square Root of 763958207163?

E 2

76395

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$$\begin{array}{r}
 763958207163 \quad (874047,027 \\
 \underline{64} \\
 167) \quad \underline{1239} \\
 \quad \quad \underline{1169} \\
 \quad \quad \quad 1744) \underline{7058} \\
 \quad \quad \quad \quad \underline{6976} \\
 \quad \quad \quad \quad \quad 174804) \underline{822071} \\
 \quad \quad \quad \quad \quad \quad \underline{699216} \\
 \quad \quad \quad \quad \quad \quad \quad 1748087) \underline{12285563} \\
 \quad \quad \quad \quad \quad \quad \quad \quad \underline{12236609} \\
 \quad \quad \quad \quad \quad \quad \quad \quad \quad 174809402) \underline{489540000} \\
 \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \underline{349618804} \\
 \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad 1748094047) \underline{13992119600} \\
 \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \underline{12236658329} \\
 \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad 1755461281, \text{ \&c.}
 \end{array}$$

IN this Example after having brought down all the Figures in the given Number, to the Remainder I bring down Cyphers, and proceed with the Work in the same Manner as if they had been Numbers given; which Work I have continued till having got three Places in Decimals in the Root, I desist, as thinking that sufficient: And here observe, that if after having extracted the Root of any given Number, there be a Remainder, then, tho' you continue the Work on ever so far, there will ever be a Remainder. For no Number whatever [whose right hand Figure is one of the 9 Digits] will produce by being squared, a Number, whose right hand Place will be a Cypher.

You may always determine as soon as you have pointed the given Number, how many Figures you will have in whole Numbers in the Root; for the Places of whole Numbers in the Root, is always equal

of MENSURATION. 41

equal to the Number of Points in the given whole Numbers, and the same for Decimals.

More EXAMPLES.

WHAT is the square Root of

$$\begin{array}{r} 36372961 \\ 64681024 \\ \quad 1,0609 \\ 911236798,794365 \end{array} \quad \left. \begin{array}{c} \} \\ \} \\ \} \\ \} \end{array} \right\} \begin{array}{c} \text{ANSWER.} \\ 6031 \\ 4968 \\ 1,03 \\ 30186,699 \end{array}$$

THESE Examples are operated in the same Manner as the preceding ones.

THERE are many Uses to which the square Root may be applied, but in this Place I shall only mention one, *viz.*

Two Numbers being given, between them to find a mean Proportional.

R U L E.

MULTIPLY the two Numbers together, and out of the Product extract the square Root, which Root is the mean Proportional required.

EXAMPLE I.

WHAT is the mean Proportional between 3 and 12; 3 multiplied by 12, is 36, whose square Root is 6, the Mean required: For as 3 is to 6, so is 6 to 12, by the Rules of Proportion.

EXAMPLE II.

FIND a mean Proportional between 4276 and 842.

E 3

4276

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$$\begin{array}{r}
 4276 \\
 842 \\
 \hline
 8552 \\
 17104 \\
 34208 \\
 \hline
 3600392
 \end{array}
 \qquad
 \begin{array}{r}
 3600392 \text{ (1897,4, \&c.)} \\
 1 \\
 \hline
 28 \overline{) 260} \\
 224 \\
 \hline
 369 \overline{) 3603} \\
 3321 \\
 \hline
 3787 \overline{) 28292} \\
 26509 \\
 \hline
 3794,4 \overline{) 178300} \\
 151776 \\
 \hline
 26524, \&c.
 \end{array}$$

So 1897,4, &c. is the mean Proportional required.

SECTION VIII.

EXTRACTION of the CUBE ROOT.

IF the finding such a Number out of a Number given, as being multiplied by itself, and this Product multiplied by the same Number, shall produce a Product either equal, or nearly so, to the Number given.

THEREFORE a Cube Number is produced by multiplying three equal Numbers together. Thus 27 is a Cube Number, produced by multiplying 3 by

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by 3, and the Product 9 by 3 again. Also 1728 is a Cube Number produced by multiplying 12 by 12, and the Product 144 by 12 again.

THE following Table exhibits the Roots and Cubes of the single Numbers.

ROOTS. 1, 2, 3, 4, 5, 6, 7, 8, 9:

CUBE. 1, 4, 27, 64, 125, 216, 343, 512, 729.

WHEN any Number is to have the Cube Root extracted, first point it into Periods of three Figures each, beginning at Units, and as many Points as there are, so many Figures must be in the Root.

THEN seek the greatest Cube in the first Period to the left Hand, whose Root place in the Quotient, and the Cube of this Root under the Period out of which it was taken, from the Period subtract the Cube, and to the Remainder bring down the next Period and call this the Resolvend, under which draw a Line; then triple the Square of the Root, that is, multiply it by three, and this triple Square set under the Resolvend, so that the triple-Square's Units may stand under the Resolvend's Hundreds; then triple the Root, and set it so, that it's Tens may stand under the triple Square's Units: Add the triple Square and triple Root together, and their Sum call a *Divisor*, and inquire how often it may be had in the Resolvend [excepting it's right hand Figure], and the Number resulting place in the Quotient, and under the Divisor draw a Line.

THEN multiply the triple Square by the last Figure placed in the Quotient, setting the Units of the Product under the Units of the triple Square: Also, multiply the triple Root by the Square of the last quotient Figure, setting the Units of the Product

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duct under the Units of the triple Root: Then set the Cube of the last Quotient Figure so, that Tens in the said Cube may stand under Units in the foregoing Line: Add these three last Lines together, and their Sum subtract from the Resolvend, to the Remainder bring down the next Period, if any, for a new Resolvend; and proceed for a Divisor, &c. as before.

EXAMPLE I

WHAT is the Cube Root of 48228544?

| | | |
|---------------|----------------|--------------------------------------|
| 48228544 (364 | | |
| | 27 | |
| | <u>21228</u> | Resolvend. |
| add { | 27 | Triple Square of 3. |
| | 09 | Triple of 3. |
| | <u>279</u> | Divisor. |
| add { | 162 | Triple Square of 3 multiplied by 6. |
| | 324 | Triple of 3 multipl. by Square of 6. |
| | <u>216</u> | Cube of 6. |
| | 19656 | Subtrahend. |
| | <u>1572544</u> | Resolvend. |
| add { | 3888 | Triple Square of 36. |
| | 108 | Triple of 36. |
| | <u>38988</u> | Divisor. |
| add { | 15552 | Triple Square of 36 mult. by 4. |
| | 1728 | Trip. of 36 mult. by Sqr. of 4. |
| | <u>64</u> | Cube of 4 Subtracted. |
| | 1572544 | |

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IF the Operation of this Example be well considered, and compared with the foregoing Rule, it will be easy to conceive how any other Example of the like Nature may be wrought ; and here observe that when the Cube Root is extracted to more than two Places, there is a Necessity of doing some Work upon a spare piece of Paper, in Order to come at the Root's triple Square, and the Product of the triple Root by the Square of the quotient Figure, &c.

IN this Example the given Number is a Cube Number, and therefore at the End of the Operation there remained nothing ; for 364 multiplied by 364, and the Product multiplied by 364 gives 48228544, the given Number.

BUT if the Number given be not a Cube Number, then to the last Remainder always bring down three Cyphers, and work anew for a decimal Fraction if needful.

More EXAMPLES.

WHAT is the Cube Root of

| | | | | |
|--------------|---|----------|---|------|
| 389017 | } | ANSWERS. | { | 73 |
| 4092727 | | | | 103 |
| 27054036008 | | | | 3002 |
| 219365327791 | | | | 6031 |
| 122615327232 | | | | 4968 |

THESE Examples are all operated in the same Manner as the foregoing one.

THERE are many Uses of the Cube Root, but I shall only mention one here, viz. To find two mean Proportionals between two given Numbers.

RULE

R U L E.

DIVIDE the greater Extream by the lesser, and the Cube Root of the Quotient multiply by the lesser Extream, and the Product is the lesser Mean. Multiply the said Cube Root by the lesser Mean, and the Product is the greater mean Proportional.

Note. THIS is only understood of those Numbers that are in continued Geometric Proportion.

E X A M P L E I.

WHAT are the two mean Proportionals between 4 and 108 ?

108 divided by 4 gives 27, whose Cube Root is 3 ; and the lesser Extream 4, multiply'd thereby, gives 12 for the lesser Mean ; and 12 multiplied by the said Root 3, gives 36 for the greater Mean.

For as 4 is to 12, so is 36 to 108.

E X A M P L E II.

FIND the two Geometric Means between 8 and 1728 ?

8) 1728 (216, whose Cube Root is 6. And 6 times 8 is 48, the lesser Mean, and 6 times 48 is 288 the greater Mean.

For as 8 is to 48, so is 288 to 1728.

WHEN any given Number is multiplied into itself any Number of times, the Product is called a *Power* ; and is named according to the Number of Times the given Number, or Root, is so multiplied.

THUS

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THUS 3 multiplied by 3 gives 9, which is the *Square*, or second Power of 3; and 3 by 3, and this Product by 3 gives 27, which is the *Cube*, or third Power of 3; and 3 by 3, and this Product by 3, and this Product by 3, gives 81, which is the *Biquadrat*, or fourth Power of 3; and 3 by 3, and this Product by 3, and this Product by 3, and this Product by 3, gives 243, which is the *Surfsolid*, or fifth Power of 3.—And the like of any other Number given: And this sort of Multiplication of any given Number is called *Involution*.

Note. THE Power any Number is raised to, is called it's *Dimension*, and the Number given to be raised is called the *Root* or *Side*, or *First Power*, or *First Dimension*.

EXAMPLE.

SUPPOSE it was required to involve the Number 6 four times; that is, to raise it to the fourth Power, or fourth Dimension?

THEN 6 by 6 is 36, the *Square* or second Power; and 36 by 6 is 216, the *Cube* or third Power; and 216 by 6 is 1296, the *Biquadrat* or fourth Power.

THERE are particular Rules for extracting the Root out of a Number of any given Dimensions; but no Extraction farther than the Cube Root being necessary in Mensuration, I have omitted all above, and for which refer the more inquisitive Reader to *Ward's young Mathematicians Guide*, where he will find enough of that Business well treated off. But as the foregoing common Rule which is given in other Books of Mensuration, is very tedious and operose: I shall before I conclude this Section give a Rule for extracting the Cube Root taken from a Series of the afore said *Ward's* Book; which is thus:

POINT

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POINT the given Number, and seek the greatest Cube in the first Period to the left Hand, whose Root place as a Quotient as before directed; and subtract the foresaid Cube from the foresaid Period, and to the Remainder bring down all the remaining Figures in the given Number; then to the Root now found annex as many Cyphers as there are remaining Points over the given Number; and this Result multiply by 3, (the Cube's Dimension) by which Product divide the Difference between the given Number, and the aforesaid Cube; and the Quotient point anew as in the Square; that is, begin at the Place of Units, and point every other Figure to the left and right [observing that the Number of Points over the Integers of this Quotient be no more than the Points brought down to the Difference between the first Period and the aforesaid Cube]. Then make the Root first found a Divisor, and inquire how often it may be had in the first Period of this Quotient, [excepting the Figure under the Point], and the Figure resulting place as a Quotient, and also on the right Hand of the Divisor; then multiply this increased Divisor by the quotient Figure, and the Product subtract from the Period out of which this last quotient Figure was taken, and to the Remainder bring down the next Period (if any) and this divide by the last increased Divisor, &c.

AN Example or two will make this Rule very intelligible.

EXAMPLE I.

WHAT is the Cube Root of 12812904?

28211,

of MENSURATION. 49

$$\begin{array}{r}
 12812904 \quad (200 \\
 \underline{8} \quad \quad \quad 3 \\
 6,00) 4812904 \quad 600 \\
 23) 8021,5 \text{ \&c.} \quad (34 \\
 \underline{69} \quad \quad \quad 200 \\
 234) 1121 \quad \quad \quad 234 \\
 \underline{936} \\
 185
 \end{array}$$

I FIRST begin at 4 the Place of Units, over which I put a Point, and omitting two Figures, I put another Point over the 2, and omitting two Figures more, I put another Point over the left hand Figure 2. Now here are 3 Points, and therefore I know I must have three Places of Integers in the Root. Then beginning with the first Period 12, I find the greatest Cube therein is 8, whose Root is 2; so I place 8 under 12, and 2 as a Quotient. Then I subtract 8 from 12, and to the Remainder 4 I bring down the remaining Figures (which occupy the Places of two Points or Periods.) Then to the Quotient (2) I annex two Cyphers, for the two Points remaining over the given Number [for the Quotient 2 is in Reality 200, because it possesses the left hand Place (*i. e.* the higheft) of the three Integers, which is the Place of hundreds,] and this 200 I multiply by 3 (the Cube's Dimension) and the Product 600 I make a Divisor, by which I divide 4812904, which is the Difference between 8 the Cube of 2, and the given Number, and the Quotient is 8021,5, &c. Then I begin at 1, the Place of Units, and point as directed in the Square, till I have put as many Points as I annexed Cyphers

F

to

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to the first Root, which in this Example is two. Then to the right and left Hands of this Number viz. 8021, &c. I draw curved Lines, as in Division, and make the first Root 2 a Divisor, inquiring how oft it may be found in the first Period 80 [excepting the Place under the Point, that is, say how often 2 in 8,] and it gives 3, which I place in the Quotient, and also, on the right Hand of the Divisor 2, which now becomes 23. This 23 I multiply by the Quotient 3, and the Product 69 I subtract from 80, and to the Remainder 11, I bring down the next Period 21, which makes 1121. Now 23 is the Divisor, which in 112 (omitting the right hand Place 1) goes 4, which 4 I place in the Quotient, and on the right Hand of 23, which now becomes 234, this 234 multiplied by the Quotient 4, gives 936, and subtracting, there remains 187.

Now this Quotient 34 added to the first Root 200, makes 234, and if this 234 be cubed, it will be 12812904, which was the Number first given, and therefore I conclude 234 to be the true Root required.

EXAMPLE II

EXTRACT the Cube Root out of 92398647506217, so that the Root may consist of eight Figures or Places.

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$$\begin{array}{r}
 92398647506217 \quad (40000 \\
 \underline{64} \quad \underline{3} \\
 12,0000) 2839864750.6217 \quad 120000 \\
 45) 236655395 \quad (52 \\
 \underline{225} \quad \underline{400, \text{ \&c.}} \\
 452) 1165 \quad 452, \text{ \&c.} \\
 \underline{904}
 \end{array}$$

THEN 452 cubed is 92345408. And

$$\begin{array}{r}
 92398647506217 \\
 \underline{92345408 \text{ subtract}} \\
 53239506217
 \end{array}$$

$$\begin{array}{r}
 92398647506217 \quad (45200 \\
 \underline{92345408} \quad \underline{3} \\
 \quad \quad 135600 \\
 1356,00) 532395062,17 \quad (39621,727293 \\
 45208) 392621,727293 \quad (08,684, \text{ \&c.} \\
 \underline{361664}
 \end{array}$$

$$\begin{array}{r}
 452086) 3095772 \quad 45200 \\
 \underline{2712516} \quad \underline{08,684} \\
 4520868) 38325672 \quad 45208,684 \\
 \underline{36166944} \quad \text{The Root.}
 \end{array}$$

$$\begin{array}{r}
 45208684) 215872893 \\
 \underline{180834736} \\
 35038157, \text{ \&c.}
 \end{array}$$

IN this Example having found 3 Places in the Root as before directed, I desist finding more till I have made a new Involution. Therefore I cube the Root 452; which gives 92345408, which used as the first Cube of 4 was, viz. subtracted from

F 2

the

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the given Number 92398647506217, leaves 53239506217, which divided by the Root 452 with two Cyphers annexed, (for the remaining two Points over the Number given,) *viz.* 45200, gives 392621,727293, which pointed as in the Square, and the Root 452 used as a Divisor, &c. as before directed, gives 08,684, which added to the foregoing Root 45200, gives 45208,684 for the Root required.

IT is found by Experience, that this Method very seldom fails giving the Root of any Number true to three Places at least, at the first operation; and if the second Place in the Root is possessed by a Cypher, or an Unit, you may get four or five Places true at the first Operation frequently.

BUT if you make a second Operation, as in the foregoing Example, you may depend on having the Root true to nine Places; but if more Accuracy be required, and you make a third Operation, you will have the Root true to 27 Places, each Operation tripling the Figures found in the last Root.

SECTION IX.

Of CHARACTERS and their EXPLANATION.

IN the following Sheets of this Treatise, there are several Characters and Expressions used in order to shorten the Work which I shall here explain.

I. WHEREVER you meet with this Sign +, (*more*) it signifies that the Number following the Sign is to be added to the Number going before it; thus 4 + 8, is read 4 more 8, and signifies, that 8 is to be added to 4.

II. THIS

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II. THIS Sign $-$ (*less*) signifies that the Number following it, is to be subtracted from the Number going before it; thus $6 - 2$ is read 6 less 2, and signifies, that 2 is to be taken from 6.

III. THIS Sign \times (*into*) signifies Multiplication, and implies that the Numbers this Sign is between, are to be multiplied together; thus 4×9 imports, that 4 is to be multiplied by 9; and $2 \times 3 \times 6 \times 5$, signifies that 2 is to be multiplied by 3, and that Product by 6, and this Product by 5, and the like of any other.

IV. THIS Sign \div (*by*) signifies Division, and shews that the Number going before the Sign, is to be divided by the Number following it; thus $12 \div 4$ implies, that 12 is to be divided by 4; but Division is most commonly expressed by setting down the Dividend or Number to be divided, and placing the Divisor or dividing Number under it, with a Line drawn between them, like a vulgar Fraction $\frac{12}{4}$ implies that 12 is to be divided by 4, and if 48,327 was to be divided by 2,15, I express it thus $\frac{48,327}{2,15}$ &c.

V. THIS Sign $=$ (*equal*) signifies that the Numbers or Expressions on each side thereof are equal one to the other; thus $4 + 8 = 12$, signifies that 8 added to 4 is equal to 12; and $6 - 2 = 4$ implies that 2 taken from 6 leaves 4, or 6 lessened by 2 is equal to 4; and $4 \times 9 = 36$ implies that 4 multiplied by 9, gives a Product equal to 36; and $\frac{12}{4} = 3$, signifies that 12 divided by 4, gives a Quotient equal to 3, and the like of other Expressions.

VI. THE Terms of Proportions are expressed by certain Points between the Terms; thus $4 : 6$

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: : 10 : 15, and is read, as 4 is to 6, so is 10 to 15, so that the two Points : between the two first Terms is read, *is to* ; the four Points : : between the second and third Terms is read, *so is*, and the two Points : between the third and fourth Terms is read, *so*.

TAKE an Example where all the forementioned Characters are used : Suppose I buy 120 Eggs at two a Penny, and 120 more at three a Penny, and sell them again at five for two Pence ; whether do I lose or gain, and how much ?

$\frac{120}{2} = 60$ *d.* the Price the first 120 cost, and $\frac{120}{3} = 40$ *d.* the Price that the second 120 cost ; and $60 + 40 = 100$, the Price the 240 cost. Then as 5 egg. : 2 *d.* : : 240 egg. : 96 *d.* ; for $240 \times 2 = 480$, and $\frac{480}{5} = 96$ *d.*, the Price the Eggs were sold at ; and 100 *d.* — 96 *d.* = 4 *d.* the Money lost.

WHEN several Terms are connected together by a Line drawn over them, it implies that the Result of those Terms, being ordered, or operated, as the Signs between them denote, is to be taken for the succeeding part of the Work ; for Example, suppose an Expression appeared thus, $6 \times 2 - 4$, it implies that 6 is to be multiplied by 2, and from the Product 12, 4 is to be taken.

Again suppose this Expression $\frac{8 \times 3 + 9}{11} - 1,5 \times 8$ occurred ; it imports, that 8 is to be multiplied by 3, and to the Product 24 is to be added 9, and the Sum 33 is to be divided by 11, and from the Quotient 3, 1,5 is to be taken, and the Remainder 1,5 is to be multiplied by 8.

AND in the same Manner other Expressions of a like kind are to be understood.

MENSURATION

MENSURATION.

PART I.

SECTION I.

MENSURATION is the Method whereby the Contents of Superficies and Solids are found, by having their Dimensions given in any sort of Measure ; and consists of three Parts ; viz. *Lineal*, *Superficial*, and *Solid*.

I. *Lineal Measure* is nothing more than the measuring of Lengths, and is used in measuring the Dimensions of superficial and solid Figures.

II. *Superficial Measure* is the finding the Number of square Inches, Feet, Yards, &c. in any flat Figure, as a Floor, a Wall, &c. by having the Dimensions in Lineal Measure.

III. *Solid Measure*, is the finding the Number of Cubic, or solid Inches, Feet, Yards, &c. in those Figures that have Length, Breadth, and Depth, as a Block of Stone, Marble, &c. by having the Dimensions in Lineal Measure.

As the chief Uses of Lineal Measure are, to take the Dimensions of superficial and solid Figures ; there need no more be said of it, than,

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12 Inches - are - 1 Foot.

3 Feet - - - - - 1 Yard.

5½ Yards, or 16½ Feet - 1 Pole, Perch or Rod.

40 Poles - - - - - 1 Furlong.

8 Furlongs - - - - - 1 Mile.

Superficial and Solid Measure I shall treat of separately; and first of

SUPERFICIAL MEASURE.

WHEREIN it will be necessary to lay down, First, *The useful Definitions, and Problems.*

Secondly, *The common Methods of calculating used by Artificers.* And

Thirdly, *The Methods of computing the Areas of various sorts of Figures.*

SECTION II.

DEFINITIONS.

I. A LINE is Length without Breadth; and is either right [strait] when it is the shortest Distance between two Points; or curved [crooked] when it is not the shortest Distance between two Points.

II. A Superficies is a Figure which hath Length and Breadth, and is inclosed or contained between right or curved Lines.

Not,

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Note, ONE curved Line may contain a Space or Superficies; but of right Lines, less than three cannot contain a Space.

III. WHEN one Line is inclined towards another Line in such a Manner, as if either or both were continued, they would meet; then the Opening of these Lines is called, *an Angle*. Fig. 1. Pl. 1.

IV. WHEN one Line standeth so on another, as to incline to neither side; but maketh the Angles on each side equal; each of those Angles is called *a right one*; and the Line so standing on the other, is called, *a Perpendicular*, to that whereon it standeth. Fig. 2.

V. ALL three sided Figures are called, *Triangles*, but admit of the following Distinctions:

First, If the three sides are unequal, it is called *a Scalene Triangle*. Fig. 3.

Secondly, If the three sides are equal, it is called *an Equilateral Triangle*. Fig. 4.

Thirdly, If only two sides are equal, it is called *an Isosceles Triangle*. Fig. 5.

Fourthly, If it hath one right Angle, it is called *a Right-angled Triangle*. Fig. 6.

VI. ALL four sided Figures are called *Quadrilaterals*, but admit of the following Distinctions:

First, WHEN the four sides are equal, if the Angles are right ones, it is called *a Square*; fig. 7. but if the Angles are not right ones, it is called *a Rhombus*. Fig. 8.

Secondly, WHEN only two sides are equal, if the Angles are right ones, it is called *a Rectangle*; fig. 9. but if the Angles are not right ones, it is called *a Rhomboides*. Fig. 10.

Note. THESE four are called *Parallelograms*, as having their opposite Sides parallel or equidistant

to.

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to each other ; but all other four sided Figures are called *Trapezia*. *Fig. 11.*

VII. A **CIRCLE** is a plain Figure, contained under, or bounded by one Line called *the Circumference* ; to which all right Lines drawn from a certain Point within the Figure, called *it's Center*, are equal. *Fig. 12.*

VIII. **THE Diameter** of a Circle is a right Line drawn through the Center, and terminated at each End by the Circumference, and divides the Circle into two equal Parts, each called *a Semicircle*.——*Note*, Half the Diameter is called *the Radius*. *Fig. 12.*

IX. **THE Chord** of a Circle is a right Line drawn within the Figure, not through the Center, and terminated at each End by the Circumference, and divides the Circle into two unequal parts, called *Segments*. *Fig. 12.*

X. If the Chord of a Circle cut the Diameter at right angles, that Part of the Diameter lying between the Chord and Circumference is called *a versed Sine*, and is the height of the Segment. *Fig. 12.*

XI. A **SECTOR** is a Figure contained under two Radii of a Circle, and that part of the Circumference included between the two Radii. *Fig. 12.*

XII. A **POLYGON** is a Figure contained under many Sides ; if a Circle will pass through all it's Angles, it is called *a regular Polygon*, otherwise *an irregular one*.——*Note*, a Polygon is named according to the Number of Sides it hath ; viz. If it has 5 Sides, it is called *a Pentagon* ; if 6 Sides, *a Hexagon* ; if 7, an *Heptagon* ; if 8, an *Octagon* ; if 9, a *Nonagon* ; if 10, a *Decagon* ; if 11, an *Undecagon* ; if 12, a *Dodecagon*. See *Fig. 13, 14, 15, 16, 17, 18, 19, 20.*

XIII. IN

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XIII. In any Quadrilateral, if a Line be drawn to any two opposite Angles, that Line is called a *Diagonal*. *Fig. 21.*

XIV. THE Altitude or Height of any Figure, is a Perpendicular, let fall from the highest Point of the Figure to it's Base, or Side opposite, which is the Line you suppose the Figure to stand on.

XV. THE Area of any Figure, is the superficial Content.

SECTION III.

AS there is sometimes a Necessity of letting fall a Perpendicular, in order to come at the Area of a Figure ; it is therefore convenient to know how to solve the following Problems.

PROBLEM I.

To bisect, or divide into two equal parts the Line A B.

CONSTRUCTION.

SET one Foot of the Compasses on the end B, and open the other to any convenient Distance greater than half AB, and with that Opening describe the Arch DE ; then set one Foot on A, and with the same Opening cross the former Arch in D and E ; and draw the Line DE, which will bisect the given Line AB in the Point C. *Fig. 22.*

PRO.

P R O B L E M II.

ON any Point C of a given Line AB, to erect a Perpendicular.

C O N S T R U C T I O N.

ON any convenient Point as D, out of the given Line, set one Foot of the Compasses, and extend the other to the Point C, with that Extent describe a Circle; and from the Point E, where it cuts the given Line AB, draw the Diameter EDF, from the Point C, and through the Extremity of the Diameter F, draw the Line CF, which will be perpendicular to the given Line AB, and stand on the Point C, as was required. *Fig. 23.*

P R O B L E M III.

To let fall a Perpendicular to any given Line AB, from a Point C given above the Line.

C O N S T R U C T I O N.

SET one Foot of the Compasses on the Point C, and with any convenient Opening describe the Arch DE, on the Points D and E; with the same Opening describe Arches below the Line, to cross each other in F, lay a Ruler by C and E, and draw the Line CD, which will be perpendicular to the Line AB, as was required. *Fig. 24.*

SECTION IV.

THE Mensuration of all right lin'd Figures may be obtained by the help of the two following Propositions.

PROPOSITION I.

HAVING the Length and Breadth of a right lined Parallelogram given, to find the Area thereof.

RULE.

MULTIPLY the Length by the Breadth, [or perpendicular Height] and the Rectangle [or Product] is the Answer.

PROPOSITION II.

HAVING the Base and perpendicular Height of any right lined Triangle given, to find the Area.

RULE.

MULTIPLY the Base by half the perpendicular Height, or the perpendicular Height by half the Base; and the Rectangle is the Area.

ARTIFICERS, by the help of these two Propositions compute the Area of all right lined Figures, and that by Numbers after three Methods, viz.

G

First,

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First. By Aliquot Parts.

Secondly. By Decimals. And

Thirdly. By Duodecimals.

BUT the latter of these are mostly used when the Dimensions are taken in Feet, Inches, &c.

DIFFERENT Artificers compute the Area of their Work by different Measures, viz.

First. By the Foot; as Glazing.

Secondly. By the Yard; as Painting, Plastering, Paving, &c.

Thirdly. By the Square of 100 Feet; as Flooring, Partitioning, Roofing, Tiling, &c.

Fourthly. By the Rod of 16½ Feet, whose Square is 272½, by which Bricklayers compute their Work.

THERE is a Table also useful for Land-measure, which is as follows:

LAND is best measured by the *Gunter's Chain*, which is 4 Poles long, and is divided into 100 Links, each being 7,92 Inches, and as 40 Poles in Length, and 4 in Breadth make a Statute Acre; therefore

$$625 \text{ square Links} = 1 \text{ Pole}$$

$$100000 \quad - \quad - \quad = 160 = 1 \text{ Acre} = 4840 \text{ Yards}$$

PROPOSITION II.

SECTION V.

QUESTION I.

WHAT is the Area of a Window, whose Length is 14 Feet 6 Inches, and Breadth 4 Feet 9 Inches?

THIS Example I shall exemplify by the three Methods of Operation.

First.

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| First. By Aliquot | Secondly. By | Thirdly. By |
|------------------------|--------------|--------------|
| In. Parts. | Decimals. | Duodecimals. |
| 6 $\frac{1}{2}$ 14 - 6 | 14.5 | 14 - 6 |
| 4 58 - 0 | 4.75 | 4 - 9 |
| 3 $\frac{1}{2}$ 71 - 3 | 10.15 | 10 - 10 - 6 |
| 3 - 7 $\frac{1}{2}$ | 580 | 68 - 10 - 6 |
| 68 - 10 $\frac{1}{2}$ | 68.875 | |

In all these different Methods of Operation, the Result turns out the same exactly; which is 68 square Feet, 10 $\frac{1}{2}$ square Inches.

BUT the method by Duodecimals being mostly in Use among Workmen, I shall shew how it is performed: The other Methods being obvious from common Arithmetic.

DUODECIMALS has it's Name from *Duodecem* Latin for 12] and by it the several Denominations of Measure are supposed to be divided each into twelve Parts.

Therefore 1 Foot = 12 Inches.

1 Inch = 12 Parts,

1 Part = 12 Seconds, &c.

BUT as all Superficies are composed or made up of the Squares of the smallest Denomination in one Dimension, as often repeated as are the Number of Parts of the like Denomination in the other Dimension.

Therefore, 1 square Foot = 144 square Inches,

1 square Inch = 144 square Parts,

1 square Part = 144 square Seconds, &c.

AND hence comes the Rule for Duodecimals, viz.

G 2

Feet

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| | | | | | |
|-------------------------|---|------------------|---|--------|---|
| Feet multiplied by Feet | | | | | |
| Feet | - | D ^o . | - | Inches | } Pro- duces { |
| Feet | - | D ^o . | - | Parts | |
| Inches | - | D ^o . | - | Inches | |
| Inches | - | D ^o . | - | Parts | |
| Parts | - | D ^o . | - | Parts | |
| | | | | | Feet, Inches, Parts, &c. Parts, Seconds, &c. Thirds, &c. |

ONLY observing, that in the Multiplication of any of the Denominations excepting Feet by Feet, if the Product be above 12, carry as many Units as there may be found 12's in that Product, to the Product of the next superior Denomination; and setting down in every Denomination (excepting Feet) so much of the Product as is under 12, or exceeds 12 or 12's.

As in the foregoing Example, viz. 14 Feet 9 Inches to be multiplied by 4 Feet 9 Inches.

| | |
|-----------|-----------|
| <i>F.</i> | <i>I.</i> |
| 14 | - 6 |
| 4 | - 9 |
| 58 | - 0 |
| 10 | - 10 - 6 |
| 68 | - 10 - 6 |

FIRST I say, 4 times 6 is 24, which are two 12's, and nothing over, therefore I set down under the Inches, and carry 2, saying 4 times 4 is 16, and 2 I carry'd is 18, which is 8 and carry 1, 4 times 1 is 4 and 1 I carry'd makes 5, which makes 58 Feet: Again, 9 times 6 is 54, which are four 12's, and 6 over, which 6 I set down under the Place of Parts, and carry 4, saying 9 times 4 makes 36, and 4 I carry'd makes 40, which is 10 and carry 4, and 9 times 1 makes 9, and 4 I carry'd makes 13, which makes this Product 130, which are ten 12's, and 10 over, therefore

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set down 10 under the Place of Inches, and as the Multiplication is ended, and nothing to carry the ten Twelves to, I set them down under the Place of Feet, and then add the Products together.

BUT here you are to note, that in the foregoing Operation, when I said 4 times 6 is 24; 24 what? Why, 24 times 12 square Inches, which is twice 144 square Inches, for so many makes 2 square Feet, which is what was carry'd to the Product of Feet. And when I said 9 times 6 is 54, that is 54 times 12 square Parts, that is 4 times 12 square Inches, and 6 times 12 square Parts; and the Product 130 is 130 times 12 square Inches, which is 10 times 144, and 10 over, that is 10 square Feet, and 10 times 12 square Inches. And the like is to be understood in other Operations. But it will be more obvious perhaps to set the lowest Denomination of the multiplying Dimension one Denomination more towards the right hand. Thus, I put the 9 Inches under the Place of Parts, and multiply by the 4 Feet as before directed.

| F. | I. | P. |
|-------|----|----|
| 2 | 1 | 1 |
| 14 | 16 | 10 |
| 4 - 9 | | |
| 58 | 0 | 0 |
| 10 | 10 | 6 |
| 68 | 10 | 6 |

Now it will often happen, that the Feet in the given Dimensions are so many, that in order to multiply them by the lesser Denominations, and take a twelfth of their Product as before directed, there is a Necessity of doing some Work upon a spare Piece of Paper; to prevent which do thus.

2 - 00 G 3 102

DIVIDE

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DIVIDE the Feet you are going to multiply with by 12, reserving the Quote and Remainder in your Mind. Then multiply the Remainder by the lesser Denominations, and set down the Overpluss of 12's under the Inches, and multiply the Quote by the same Dimension, adding to the Quote what you carry'd, and it gives the Feet.

As in the foregoing Example I say, 12 in 14 goes 1, and 2 remains; then in multiplying by the 9 Inches, I say, 9 times 6 is 54, that is 6 and go 4, setting the 6 under the Place of Parts; then 9 times 2 (the Remainder) is 18, and 4 carry'd is 22, that is 10, and go 1, set the 10 under the Inches, and carry 1; then 9 times 1 (the Quote) is 9, and 1 carry'd makes 10, which set under the Feet; and adding the several Products together, as before directed, ends the Work.

THE Use of dividing the Feet thus by 12, and working with the Quote and Remainder, will appear more conspicuous in those Operations where the Feet are many, as it will be evident by comparing one Example wrought both ways.

EXAMPLES.

WHAT will the Product be, if 368 Feet 10 Inches be multiply'd by 137 Feet 8 Inches?

In order to work this Example in the most ready manner, I operate thus:

| F. | I. | P. |
|-------|----|----|
| 368 | 10 | 0 |
| 137 | 8 | |
| 245 | 10 | 8 |
| 114 | 2 | |
| 2576 | | |
| 4784 | | |
| 50776 | | |
| | 00 | 8 |

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THE 12's in 368 are 30, and 8 remains, which Quote and Remainder I reserve in my Mind, and say, 8 times 10 is 80, that is 8 and carry 6, the 8 I place under the Parts, and say 8 times 8 (the Remainder) is 64, and 6 carry'd make 70, which is 10 and carry 5, the 10 I place under the Inches, and say, 8 times 30 (the Quote) is 240, and 5 carry'd, make 245, which I set under the Feet; and so the multiplying with the 8 Inches is done. Again, the 12's in 137 are 11, and 5 remains, which Quote and Remainder I reserve in my Mind, which multiply by the 10 Inches; saying, 10 times 5 (the Remainder) is 50, which is 2 and carry 4, the 2 I put under the Inches, and say, 10 times 11 is 110, and 4 carry'd is 114, which I set under the Feet; and so the multiplying 137 Feet by the 10 Inches is done. Lastly, I multiply the 368 Feet by the 137 Feet, setting down the Product as you see in the Example, and adding the several Products together, the Work is finished.

THERE is very seldom any Occasion for multiplying Feet, Inches, and Parts; but if at any time you meet with a Necessity for so doing, you will very easily know how, by carefully examining this Example as here wrought:

| F. | I. | P. | S. | T. |
|-------|------|------|-----|-----|
| 39 | - 10 | - 7 | - 0 | - 0 |
| | 18 | - 8 | - 4 | - 0 |
| <hr/> | | | | |
| 1 | - 1 | - 3 | - 6 | - 4 |
| 26 | - 7 | - 0 | - 8 | |
| 0 | - 10 | - 6 | | |
| 15 | - 0 | | | |
| <hr/> | | | | |
| 702 | | | | |
| <hr/> | | | | |
| 745 | - 6 | - 10 | - 2 | - 4 |

S. E. C. T.

The 1st is 308 and 8 remains, which Quote and Remainder 1 relative to my

And, and lay 8 times to 20, that is 8 and carry 6, the 8 I place under the 20, and lay

8 times to 10 and carry 2, the 10 I place under the 10, and lay 8 times to 20, that is 8 and carry 6, the 8 I place under the 20, and lay

8 times to 10 and carry 2, the 10 I place under the 10, and lay 8 times to 20, that is 8 and carry 6, the 8 I place under the 20, and lay

8 times to 10 and carry 2, the 10 I place under the 10, and lay 8 times to 20, that is 8 and carry 6, the 8 I place under the 20, and lay

8 times to 10 and carry 2, the 10 I place under the 10, and lay 8 times to 20, that is 8 and carry 6, the 8 I place under the 20, and lay

8 times to 10 and carry 2, the 10 I place under the 10, and lay 8 times to 20, that is 8 and carry 6, the 8 I place under the 20, and lay

8 times to 10 and carry 2, the 10 I place under the 10, and lay 8 times to 20, that is 8 and carry 6, the 8 I place under the 20, and lay

8 times to 10 and carry 2, the 10 I place under the 10, and lay 8 times to 20, that is 8 and carry 6, the 8 I place under the 20, and lay

8 times to 10 and carry 2, the 10 I place under the 10, and lay 8 times to 20, that is 8 and carry 6, the 8 I place under the 20, and lay

8 times to 10 and carry 2, the 10 I place under the 10, and lay 8 times to 20, that is 8 and carry 6, the 8 I place under the 20, and lay

8 times to 10 and carry 2, the 10 I place under the 10, and lay 8 times to 20, that is 8 and carry 6, the 8 I place under the 20, and lay

8 times to 10 and carry 2, the 10 I place under the 10, and lay 8 times to 20, that is 8 and carry 6, the 8 I place under the 20, and lay

8 times to 10 and carry 2, the 10 I place under the 10, and lay 8 times to 20, that is 8 and carry 6, the 8 I place under the 20, and lay

8 times to 10 and carry 2, the 10 I place under the 10, and lay 8 times to 20, that is 8 and carry 6, the 8 I place under the 20, and lay

8 times to 10 and carry 2, the 10 I place under the 10, and lay 8 times to 20, that is 8 and carry 6, the 8 I place under the 20, and lay

8 times to 10 and carry 2, the 10 I place under the 10, and lay 8 times to 20, that is 8 and carry 6, the 8 I place under the 20, and lay

8 times to 10 and carry 2, the 10 I place under the 10, and lay 8 times to 20, that is 8 and carry 6, the 8 I place under the 20, and lay

8 times to 10 and carry 2, the 10 I place under the 10, and lay 8 times to 20, that is 8 and carry 6, the 8 I place under the 20, and lay

SECTION VI.

OF ARTIFICERS WORKS.

FIRST, by the Foot; as Glazings and Masons Flat Work.

QUESTION II.

WHAT will the glazing a Triangular Skylight come to at 10 *d.* per Foot, supposing the Base 12 Feet 6 Inches long, and the perpendicular Height 10 Feet 9 Inches?

By Duodecimals.

There is very seldom any Occasion for multiplying by 12, but it is at any time necessary for finding the Area of a Triangle, you will find it necessary to multiply the Base by the Height, and divide the Product by 2.

Example as before.

| | | |
|-----------------|-------------------------------|---------------------------|
| d. & d. | $\frac{1}{2}$, $\frac{1}{4}$ | 100 |
| 6 | 4 | 104 - 8 - 3 = the Area. |
| 1 | d. | 82 |
| 6 $\frac{1}{2}$ | 10 | 34 - 8 |
| 2 $\frac{1}{2}$ | 5 | 0 - 6 $\frac{1}{2}$ |
| | 1 $\frac{1}{2}$ | 20) 8,7 - 2 $\frac{1}{2}$ |
| | 6 $\frac{1}{2}$ | 4 - 7 - 2 $\frac{1}{2}$ |

ANSWER, 4 l. 7 s. 2 $\frac{1}{2}$ d.

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By Decimals.

$$\begin{array}{r}
 16,75 \\
 \underline{6,25} \\
 8375 \\
 3350 \quad d. \quad \text{£.} \\
 10050 \quad -10 = 14010 \\
 \hline
 104,6875 \\
 \text{£ } 140,0, \text{ the Multiplier} \\
 9) 628 \quad \text{inverted.} \\
 \underline{69} \\
 1047 \\
 \underline{41875} \\
 4,36197
 \end{array}$$

If what has been already said in the Decimal Part be well understood, it will be no difficult Matter to see (immediately) how this Decimal Operation is performed, and also those which follow; the Area given by the Duodecimal Work, is valued by Aliquot Parts or Practice.

QUESTION III.

THERE is a House with three Tier of Windows, three in a Tier, the Heights are, viz. First Tier 7 Feet, 10 Inches.—Second, 6 Feet, 8 Inches.—And third 5 Feet, 4 Inches. And the Breadth of each 3 Feet, 11 Inches; what will the Glazing come to at 14 d. per Foot?

$$\begin{array}{r}
 81081 \\
 11011 \\
 \hline
 90092
 \end{array}$$

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By Duodecimals.

F. I. Decimals By
 Add $\begin{array}{r} 7 - 10 \\ 6 - 8 \\ 5 - 48 \end{array}$

Multiply by $\begin{array}{r} 10 - 10 \\ 59 - 6 \\ 3 - 11 \\ 54 - 6 - 6 \\ 178 - 6 \end{array}$

$\begin{array}{l} d. \\ 2 \end{array} \left| \frac{1}{2} \right| \begin{array}{r} 233 - 0.16 \text{ Parts.} \\ 233 \\ 38 - 10 \end{array} \begin{array}{l} 6 \text{ is } \frac{1}{2} \\ 14 d. \\ \frac{1}{2} \end{array}$

ANSWER, $\begin{array}{r} 20) 27,1 - 10\frac{1}{2} \\ 13 - 11 - 10\frac{1}{2} \end{array}$

By Decimals.

$\begin{array}{r} 59.5 \\ 3.916 \\ 9) 3570 \\ 306 \\ 510 \\ 595 \end{array}$

III

There is a House with 3 Tiers of Windows, three in a Tier, the Height of the first Tier 7 Feet, 10 Inches, 6 Feet, 8 Inches, and third 2 Feet, 10 Inches. And the Breadth of each 3 Feet, 11 Inches. What will the Glazing come to at 14 d. per Foot?

$\begin{array}{r} 5355 \\ 1785 \\ 233.0416 \\ 2850.0 \\ 9) 699 \\ 776 \\ 18643 \\ 116521 \\ 135940 \end{array}$

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To solve this Question, I first add together the Heights of the Windows over each other, and then multiply the Sum by 3 the Number of Rows, which gives a Length equal to the nine Windows set on each other, and this multiply'd by the Breadth gives the Area.

GLAZIERS generally measure their Work to a Quarter of an Inch; and never make any allowances for round or oval Windows, but always measure them to the greatest Length; for there is more Trouble in cutting the Glass to those Shapes, than the Value of the Glass omitted.

QUESTION IV.

WHAT is a Marble Slab worth, whose Length is 5 Feet 7 Inches, and Breadth 1 Foot 10 Inches, at 6 s. per Foot?

| By Duodecimals. | | By Decimals. |
|-----------------|--------|--------------|
| 5 | 7 - 0 | 5,583 |
| | 1 - 10 | 1,83 |
| 4 | 7 - 10 | 9) 16750 |
| 5 | 7 | 18611 |
| 10 | 2 - 10 | 44666 |
| 5 | 2 - 10 | 55833 |
| 10 | 10 | 10,2361 |
| 1 | 5 | .3 = 6 |
| 3 | 01 - 5 | 3,07083 |
| ANSWER, | | |
| 34 1 s. 5 d. | | |

Secondly.

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Secondly. By the Yard; as Paviers, Painters, Plaiſſiers, and Joyners.

R U L E.

DIVIDE the Feet (found as before) by 9 [because 9 Square Feet are 1 Yard] and it gives the Area.

QUESTION V.

WHAT will the paving a Yard of a rectangular Form come to at 3 s. 2 d per Yard; supposing the Length 27 Feet, 10 Inches, and the Breadth 14 Feet, 9 Inches?

By Duodecimals.

| | | | | | | | |
|--|--|--|--|----------------------------|--|----------|--|
| | | | | 27 - 10 | | | |
| | | | | 14 - 9 | | | |
| | | | | <u>20 - 10 - 6</u> | | | |
| | | | | 11 - 8 | | | |
| | | | | 387 | | | |
| | | | | 9) 410 - 6 - 6 | | | |
| | | | | <u>45 - 5 - 6</u> | | 6 at 3 - | |
| | | | | 45 | | | |
| | | | | <u>4 - 10</u> | | | |
| | | | | 2 - 5 | | | |
| | | | | 7 - 6 | | | |
| | | | | <u>1 - 10 1/2</u> | | | |
| | | | | 7 - 4 - 4 1/2 | | | |
| | | | | 4 1/2 | | | |
| | | | | 4 1/2 | | | |
| | | | | 2 | | | |
| | | | | <u>1 - 10 1/2</u> | | | |
| | | | | ANSWER, 7 l. 4 s. 4 1/2 d. | | | |

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By Decimals.

$$\begin{array}{r}
 27,83 \\
 \underline{14,74} \\
 13916 \\
 194833 \quad s. \quad d. \\
 1113333 \quad 3 - 2 = ,1583 \\
 \underline{2783333} \\
 9) 410,5416 \\
 \underline{45,615146} \\
 8851,0 \\
 9) 137 \\
 \underline{152} \\
 3649 \\
 22808 \\
 \underline{45616} \\
 7,2225
 \end{array}$$

QUESTION VI.

I HAVE paved a rectangular Court-Yard 42 Feet, 9 Inches in Front; and 68 Feet, 6 Inches in Depth: And in this I have laid a Foot-Way the Depth of the Court, of 5 Feet, 6 Inches in Breadth: The Foot-Way is laid with Purbeck Stone at 3 s. 6 d. per Yard, and the Rest with Pebbles at 3 s. per Yard; what will the whole come to?

Find the Value of the whole Court at 3 s. and the Foot-Way at 6 d. and these Values added together will give the whole Cost.

QUESTION VII.

WHAT will the plaistering a Ceiling at 10 d. per Yard come to; supposing the Length 21 Feet, 8 Inches, and the Breadth 14 Feet, 10 Inches?

By Duodecimals.

$$\begin{array}{r}
 21 - 8 \\
 14 - 10 \\
 \hline
 18 - 0 - 8 \\
 9 - 4 \\
 \hline
 294
 \end{array}$$

$$\begin{array}{r}
 d. 8 \quad d. 89 \quad 321 - 4 - 8 \quad d. \\
 6 \text{ and } 4 \quad \frac{1}{2}, \frac{1}{3} \quad 35 - 6 - 4 - 8 \text{ at } 10
 \end{array}$$

$$\begin{array}{r}
 17 - 6 \\
 11 - 8 \\
 \hline
 6\frac{1}{2}
 \end{array}$$

$$\begin{array}{r}
 20 \quad 29 - 8\frac{1}{2} \\
 \hline
 1 - 9 - 8\frac{1}{2}
 \end{array}$$

ANSW. 1 l. 9 s. 8½ d.

$$\begin{array}{r}
 3\frac{1}{2} \\
 3\frac{1}{2} \\
 \hline
 6\frac{1}{2}
 \end{array}$$

H 2

By

By Decimals.

$$\begin{array}{r}
 14,87 \\
 \underline{21,6} \\
 9) 8900 \\
 \underline{9188} \\
 1483 \text{ d.} \\
 29666 \text{ f.} \\
 10 = ,0416
 \end{array}$$

PLASTERERS Works are principally of two Kinds, namely, *first*, Works lath'd and plaister'd which is call'd *Ceiling*. *Secondly*, Works rendered which is of two Kinds, *viz.* upon Brick Wall or between Quarters in the Partition between Rooms.

IN measuring Rendering upon Brick Walls, there are no Deductions made ; but in measuring Rendering between Quarters, there is commonly a fifth Part of the whole Area deducted : But when Rendering between Quarters is whited or coloured there is commonly a fourth or fifth Part added to the whole Area, for the Sides of the Quarters and Braces, &c.

*Note, You must make Deductions for Doors
Windows, &c.*

QUESTION

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QUESTION VIII.

THERE is a Quantity of Partitioning that measures 234 Feet, 8 Inches about, and 14 Feet, 6 Inches high; but is Rendered between Quarters: The Lathing and Plaistering will be 8 d. per Yard, and the Whiting 2 d. per Yard; what will the whole come to?

By Duodecimals.

234 - 8

14 - 6

577 - 4 - 0

9 - 4

3276

3402 - 8

5) 378 - 0 - 8

75 - 5 - 6

302 - 4 - 2

378 - 0 - 8

75 - 5 - 6

453 - 6 - 2

By Decimals.

234.6

14.5

117.33

93866

234666

9) 3402.6

5) 378.074

75.6148

302.4592

378.074

75.6148

453.688

The Area in Yards.

This $\frac{1}{2}$ Part deduct.

Rem. the Ar. plaister'd.

To the Area in Yards

Add the $\frac{1}{2}$ Part.

Gives the Ar. whited.

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$$\begin{array}{r} 302,4592 \\ 8 d. = ,02 l. \end{array}$$

$$\underline{10,081973}$$

l. s. d.

$$\underline{10-1-7\frac{1}{2}}$$

$$\begin{array}{r} 453,6888 \\ 2 d. = ,0082 l. \end{array}$$

$$\underline{1512296}$$

$$\underline{36295104}$$

$$\underline{3,64463336}$$

l. s. d.

$$\underline{3-12-10\frac{1}{2}}$$

$$10,081973$$

$$\underline{3,64463336}$$

$$\underline{13,72660636}$$

ANSWER, 13 *l.* 14 *s.* 6 $\frac{1}{2}$ *d.* the whole Cost.

IN these two Operations, instead of multiplying by 3, and dividing by 9, as directed; I take $\frac{1}{3}$ of the Multiplicand, which is exactly the same, tho' more expeditious.

QUESTION. EX.

SUPPOSE a Room: that was painted at 8 *d.* per Yard, measures as follows: The Height (taking in the Cornice and Mouldings) is 11 Feet, 7 Inches; the Girt or Compass 74 Feet, 10 Inches; the Door 7 Feet, 6 Inches, by 3 Feet, 9 Inches; five Window-Shutters, each 6 Feet, 8 Inches, by 3 Feet, 4 Inches; the Breaks in the Windows 14 Inches deep; and 8 high; the Chimney 6 Feet, 9 Inches by 5 Feet; a Closet the Height of the Room 3 $\frac{1}{2}$ Feet deep, and 4 $\frac{1}{2}$ Feet in Front, with Shelving together 22 Feet, 6 Inches by 10 Inches; the Shutter, Door, and Shelves, are coloured on both Sides: What will the whole come to?

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$$7488 \times 11588 =$$

866,7308

The Area of the whole Room.

$$68 \times 38 \times 5 =$$

111,111

The Area of the Shutters once.

$$8 + 8 + 38 + 38 = 228 \times 1118 \times 5 = 132,422$$

The Breaks in the Windows.

$$75 \times 375 =$$

28,125

The Door once.

$$35 \times 2 + 475 \times 2 = 165 \times 11,588 = 191,125$$

The Closets Area.

$$225 \times 88 \times 2 =$$

375

The Area of the Shelves.

The Sum of the Areas

1366,8138

Out of which deduct.

$$675 \times 5 =$$

3375

The Chimney's Area.

Remains

1333,0638

The Area of the whole Work.

Then $\frac{1333,0638}{9} = 148,1182$ Yds. &c. then
if 1 yd. : 08 l. :: 148,1182 yd. : 4937 27 l. &c.
= 4 l. 18 s. 9 d.

PAINTERS

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PAINTERS take their Dimensions with a String, and measure from the Top of the Cornice to the Floor, girting the String over all the Mouldings and swelling Pannels : And in measuring of Doors, they account the Height and Breadth of the Door so much more, as in the thickness of the Stuff ; it being reasonable they should be paid for all Places whereon their Colour is laid. Their Price they generally proportion according to the Number of Times they lay their Colour on.

Note, There must always Deductions be made for Chimnies, Casements, &c. if any within the Dimensions they take.

QUESTION X.

WHAT will the Wainscoting a Room come to at 6 s. per square Yard ; supposing the Height of the Room (taking in the Cornice and Mouldings) is 12 Feet 6 Inches, and the Compass is 82 Feet 8 Inches ; the Window Shutters each 7 Feet 8 Inches, by 3 Feet 6 Inches, and the Door 7 Feet by 3 Feet 6 Inches ; the Shutters and Door being worked on both Sides is reckoned Work, and half Work ?

F.T.P.

of MENSURATION. 81

1004
41.10
41.10
41.10

$$\begin{array}{r} F. 1. P. \\ 83 - 8 - 0 \\ \hline 12 - 6 \end{array}$$

$$\begin{array}{r} F. 1. P. \\ 7 - 8 \\ \hline 3 - 6 \end{array}$$

$$\begin{array}{r} F. 1. P. \\ 3 - 6 \\ \hline 7 \end{array}$$

$$\begin{array}{r} 41 - 10 - 0 \\ \hline 104 - 00 \end{array}$$

$$\begin{array}{r} 3 - 10 - 0 \\ \hline 23 - 0 \end{array}$$

$$\begin{array}{r} 24 - 6 \text{ the Door once,} \\ \hline 80 - 6 \text{ the Shutters once.} \end{array}$$

Add { 145 - 10 Room's Area. 26 - 10

52 - 6 = $\frac{1}{2}$ Do. & Sh. D^o. 3 Windows

$$\begin{array}{r} 2) 105 - 0 \\ \hline 52 - 6 \text{ the } \frac{1}{2} \text{ Door and Shutt}$$

Sum 198 - 4 = whole W.

$$\begin{array}{r} 80 - 6 = \text{Shut. once} \end{array}$$

$$\begin{array}{r} F. 1. P. \\ 4 = \frac{1}{2} \end{array}$$

F. 1. yds. F. 1.

$$\begin{array}{r} 22 \\ \hline 3 = 6 s. \end{array}$$

$$\begin{array}{r} 4 = \frac{1}{2} \end{array}$$

9) 198 - 4 (22 - 0 + 4 Then {

$$\begin{array}{r} 6 - 12 \\ \hline 6 - 12 \end{array}$$

$$\begin{array}{r} 1. s. d. \\ 6 12 2\frac{1}{2} \end{array}$$

$$\begin{array}{r} 6 - 12 - 2\frac{1}{2} \end{array}$$

ANSWER, 6 12 2 $\frac{1}{2}$

JOYNER

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JOYNNERS measure their Work in Height with a String, and their Length or Compass upon the Floor as Painters do; for they say, they ought to measure where their Plane touches, therefore they take the Cornice and Mouldings into the Height of the Room.

Note. THEY only take the Cornice and Mouldings in with the Height of the Room, when they are struck with a Horse-plane, (as they call it) but if they are wrought by Hand, then they are paid so much *per Foot* running Measure.

ALL Stuff an Inch and half thick and under, wrought on both Sides, is by them reckoned at Work and Half Work; but Stuff of greater Thickness wrought on both Sides, is valued at Double Work.

THEY make Deduction for all Vacancies that fall within their Work; and Window Boards, Sofite Boards, Cheeks, &c. are measured by themselves.

Thirdly, By the Square: As, Flooring, Partitioning, Roofing, Tying, &c. such Works are measured by a ten Feet Rod, whose Square is 100. Therefore:

R U L E.

DIVIDE the Feet, (found as before) by 100, and it gives the Area.

Q U E S T I O N X I.

SUPPOSE a House of three Stories, besides the Ground-floor, was to be floor'd at 6 l. 10 s. *per Square*; the House measures 20 Feet 8 Inches, by 16 Feet 9 Inches; there are 7 Fire-places, whose Measures are as follows: Two each 6 Feet, by 4 Feet 6 Inches; two other, each 6 Feet, by 5 Feet 4 Inches; and two each 5 Feet 8 Inches, by 4 Feet 8 Inches; and the seventh 5 Feet 2 Inches, by 4 Feet; and the Well-Hole for the Stairs is 10 Feet 6 Inches, by 8 Feet 9 Inches; what will the Whole come to? 4 - 6

of *MENSURATION.* 83

4-6 5-4 5-8

5-1-2 10-6

$$\begin{array}{r} \overline{27} - 0 \\ \underline{32} - 0 \\ 3 - 9 - 4 \end{array}$$

$$\frac{20}{7-10-6}-8$$

$$\begin{array}{r} 54 \\ \hline 64 \\ \hline 26 - 5 - 4 \end{array}$$

91-10-6

$$\begin{array}{r} 20-8 \\ 16-0 \\ \hline 52-10-8 \end{array}$$

367 - 6 - 0 The Well Hole.

15-6-0

54 - 0 - 0 The first Chimneys.

330-8

64 - 0 - 0 The Second Chimnies.

346 - 2

52-10-8 The third Chimnies.

20 - 0 - 0 1 ne Jourdain Guinness.

1384 - 8 The Ar. of the four Floors.

559 - 0 - 8 The whole Deductions.

559 - 0 - 8 The A.R. of the Deduct.

| Sqrs. | 1. | 2. | 3. | 4. |
|-------|----|----|----|----|
| 8 | 25 | 6 | 10 | 10 |

100) 825 - 7 - 4 The Area of the Work.

11

F. I. P. I. "

67. $\frac{1}{50}$ 1-12-6

8-25-7-4 at 6-10 per Square.

[illegible]

IN

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IN Flooring there are Deductions made also for Hearth-Stones, excepting the Hearth-Stone hath a Border round it; and then the Hearth is measured in with the Floor.

QUESTION XII.

IN 173 Feet 10 Inches in Length, and 10 Feet 7 Inches in Height of Partitioning; how many Squares?

By Duodecimals.

$$\begin{array}{r}
 173 - 10 \\
 \quad 10 - 7 \\
 \hline
 101 - 4 - 10 \\
 1738 - 4 \\
 100) 1839 - 8 - 10 \\
 \text{S.} \quad \text{F.} \quad \text{I.} \quad \text{P.} \\
 18 - 39 - 8 - 10
 \end{array}$$

By Decimals.

$$\begin{array}{r}
 173,88 \\
 885,01 \left\{ \begin{array}{l} \text{Breadth} \\ \text{inverted.} \end{array} \right. \\
 9) \quad 52 \\
 \quad 58 \\
 \hline
 1390 \\
 8691 \\
 \hline
 173833 \text{ Sgrs.} \\
 100) 1839,72 (18,3973
 \end{array}$$

IN Roofing, Tyling and Slating, it is customary to reckon the flat and half of any Building within the Walls to be the Measure of the Roof of that Building, when the said Roof is of a true Pitch.

Note, ALL Roofs are said to be of a true Pitch, when the Rafters are $\frac{1}{2}$ of the Breadth of the Building.

IF the Roof is more or less than true Pitch, they measure from one side to the other with a Rod or String.

QUESTION

of MENSURATION. 85

QUESTION XIII.

If a House measure within the Walls 52 Feet, 8 Inches in Length, and 30 Feet, 6 Inches in Breadth; and supposing the Roof to be of a true Pitch; what will it cost Roofing at 10s. 6 d. per Square?

By Duodecimals.

2) 30 — 6 — The Breadth of the Building — 2) 30,5

15 — 3 — The half Breadth — 15,25

45 — 9 — The Breadth of the Roof — 45,75

52 — 8

30 — 6 — 0

39 — 0

90

225

100) 2409 — 6

Sqr. F. l. 's, d.

24 — 9 — 6 at 10 — 6 per Square.

12 — 0

— 12

— — 11½

12 — 12 — 11½

5 and 4

6

By Decimals.

2) 30,5

15,25

45,75

5,25

9) 27450

30500

9150

22875

100) 2409,5

24095 Sqr's.

525

120475

48190

120475

120475

12,649875

ANSW. 12 l. 12 s. 11½ d.

THERE

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THERE are other Works about a Building done by the Carpenter, that are measured by the Foot Running Measure; as Cornices, Doors, and Cases, Windows-Frames, Lintels, Guttering, Sommers, Skirt Boards, &c.

IN the measuring of Roofing for Workmanship alone, they generally deduct for the Holes for Chimney Shafts and Skylights, if they are any thing considerable.

BUT measuring for Work and Materials, they commonly measure in all Skylights, Lutheren Lights, and Holes for the Chimney Shafts, for their Trouble and Waste of Stuff; excepting such Skylights as are very large, that is, exceed nine or ten Feet in Area.

Q U E S T I O N X I V .

WHAT will the Tying a Barn cost me at 25 s. 6 d. per Square; the Length being 43 Feet, 10 Inches, and Breadth 27 Feet, 5 Inches on the Flat; the Eaves Boards projecting 16 Inches on each Side?

By

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By Duodecimals.

2) 27 - 5 ——— The Breadth ——— 2) 2 7,4 1 8 6
13 - 8 - 6 — The half Breadth — 1 3,7 0 8 8
2 - 8 — The Eaves Boards Doubled — 2,8 6 6 6
1 - 4 X 2 =

By Decimals.

4 3.7 9 1 6
8 8.3 4 Length inverted.

$$\begin{array}{r} 43-9-6 \\ 43-10-0 \\ \hline 36-5-11-0 \end{array}$$

9) 1314

$$\begin{array}{r} 1460 \\ 25 - 6 = 1,275 \\ \hline 35033 \end{array}$$

8975

5 - 9 - 6161 (001)

$$\begin{array}{r} 100) 1919,534 \\ \underline{100000} \\ 919534 \\ \underline{900000} \\ 19534 \\ \underline{100000} \\ 95340 \\ \underline{900000} \\ 53400 \\ \underline{500000} \\ 34000 \\ \underline{300000} \\ 40000 \\ \underline{400000} \\ 00000 \end{array}$$

S. F. I. P. J. d.
19-19-6-5 at 25-6 per square.

$$\begin{array}{r} 572.1 \\ \hline 191953 \end{array}$$
$$\begin{array}{r} 52 \\ 6 \overline{) 312} \\ \underline{30} \\ 12 \\ 12 \\ \underline{0} \end{array}$$
[illegible]
$$\begin{array}{r} 2474 \\ \hline \end{array}$$

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Note, WHEN the Ridge of a Roof does not run strait but bends into an Angle, then the Tyles coming from the Ridge to the Eaves make an Angle also; and that Angle of the Roof which bends inwards, is called a *Valley*; and that Angle that bends outwards, is called a *Hip*; and in Tyling and Slating, it is common to add the Length of the Vallies (measured from the Ridge to the Eaves) to the Content in Feet; sometimes the Hips are added.

In Slating it is common to reckon the Breadth of the Roof 2 or 3 Inches broader than what it measures, because the first Row is almost cover'd by the second; this is done sometimes when a Roof is tyed.

Note, SKYLIGHTS and Chimney Shafts are deducted, but they seldom deduct Luthern Lights (or Garret Windows on the Roof) for the covering such they reckon equal to the Hole in the Roof.

Fourthly. BY the Rod, as, Bricklayers Work.

R U L E.

DIVIDE the Area found in Fact, by the Square of $16\frac{1}{2}$ Feet, or 272 (the odd Quarter being seldom minded in Practice) and the Quote is the Rods.

Note, BRICKLAYERS always compute or value their Work at the Rate of a Brick and an half thick; and if the Thickness of the Wall happen to be more or less than such, it must be reduced to that Thickne's as follows:

MULTIPLY the Area of the Wall by the Number of half Bricks the Thickness of the Wall is of; and divide the Product by 3 (because $1\frac{1}{2}$ Brick is 3 halves) and it gives the Area.

QUESTION

of MENSURATION. 89

QUESTION XV.

How many square Rods are there in a Wall $62\frac{1}{2}$ Feet long, 14 Feet, 8 Inches high, and $2\frac{1}{2}$ Bricks thick?

By Duodecimals.

$$\begin{array}{r}
 92 - 6 \\
 \underline{14 - 8} \\
 41 - 8 - 0 \\
 875 - 0 \text{ R. P. I.} \\
 272 \overline{) 916 - 8} \quad (3 - 100 - 8 \\
 \underline{5} \\
 3 \overline{) 16 - 231 - 4} \\
 \underline{5 - 167 - 9 - 4} \\
 \text{R. P. I. P. } 5 - 167 - 9 - 4 \\
 \text{Answ. } 5 - 167 - 9 - 4
 \end{array}$$

By Decimals.

$$\begin{array}{r}
 62,5 \\
 \underline{14,8} \\
 9 \overline{) 3750} \\
 \underline{4166} \\
 8750 \text{ Rods.} \\
 272 \overline{) 916,8} \quad (3,37, \&c. \\
 \underline{5} \\
 3 \overline{) 16,85} \\
 \underline{5,616, \&c.}
 \end{array}$$

I 3

In

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IN measuring of Brickwork they are commonly very careful in Regard to their Allowances; for one Foot square in the Front is commonly worth Sixpence, so that any considerable Error would amount to too much to reject.

IN great Buildings they often deduct all the Timbers laid in the Walls, but this is only when the Workmanship is very good; for in general it is allow'd in, because they reckon the Time they generally wait on the Carpenter, together with the Bedding those Timbers in Mortar, is equal to the Brickwork that would supply the Timbers Place.

QUESTION XVI.

SUPPOSE the Side Walls of an House 28 Feet, 10 Inches in Length, and the Height of the Roof from the Ground 53 Feet, 8 Inches, and the Gable (or Triangular Part at Top) to rise 42 Course of Bricks (reckoning 4 Course to a Foot). Now 20 Feet high is $2\frac{1}{2}$ Bricks thick; 20 Feet more 2 Bricks thick; 15 Feet 8 Inches more at $1\frac{1}{2}$ Brick thick; and the Gable 1 Brick thick; what will the whole Work come to at 5 l. 16 s. per Rod?

$$\frac{42}{4} = 10,5$$

of MENSURATION. 91

$\frac{4^2}{4} = 10,5$ the Height of the Gable half is 5,25 Feet.

$$\begin{array}{r} 28,8\text{ g} \\ 20 \\ \hline 576,8 \\ 5 \end{array}$$

$$\begin{array}{r} 28,8\text{ g} \\ 20 \\ \hline 576,8 \\ 4 \end{array}$$

$$\begin{array}{r} 28,8\text{ g} \\ 15,8 \\ \hline 9)17300 \\ 192\text{ f} \\ \hline 43250 \\ 451,7\text{ f} \end{array}$$

$$\begin{array}{r} 28,8\text{ g} \\ 5,25 \\ \hline 14416 \\ 57666 \\ 1441666 \\ 151,375 \\ 2 \end{array}$$

$$\begin{array}{r} 3)302,75 \\ \hline 100,91\text{ g} \end{array}$$

F. L. F. L.
As 272 : 5,8 :: 2282,63 g : 48,67 f
L. : d.
ANSWER, 48 — 13 — 5 f.

$$\begin{array}{r} 100,91\text{ g} \text{ at } 1\text{ f} \\ 451,72\text{ f} \text{ at } 1\frac{1}{2}\text{ f} \\ 768,888 \text{ at } 2\text{ f} \\ 961,111 \text{ at } 2\frac{1}{2}\text{ f} \\ \hline 2282,63\text{ g} \end{array}$$

} Bricks
thick.

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IN all Buildings the Thickness of the Walls generally decrease as they rise; and it is usual to set off half a Brick at each Decrease: The Thickness is commonly set off on the Inside, and that in a Place where a Floor will come, so that the Set-off is thereby hid.

IT is common to build from a Base 4 Course of Bricks high, and which projects two or three Inches on each Side of the Wall.

THE different Thicknesses are measured separate, and reduced each to a Brick and an half thick, and then added together.

IF you are to measure a Chimney standing alone by itself, without any Party-Wall being adjoined, then girt it about for the Length, and the Height of the Story is the Breadth. The Thickness must be the same the Jaumbs are off, provided that the Chimney be wrought upright from the Mantle-Tree to the Ceiling, not deducting any thing for the Vacancy between the Floor (or Hearth) and the Mantle-Tree, because of the Gathering of the Breast and Wings to make Room for the Hearth in the next Story.

IF the Chimney-Back be a Party-Wall, and the Wall be measured by itself, then you must measure the Depth of the two Jaumbs, and the Length of the Breast for a Length, and the Height of the Story the Breadth, at the same Thickness your Jaumbs were off.

WHEN you measure Chymney Shafts, girt them with a Line round about the least Place of them for the Length, and the Height shall be your Breadth; and if they be four Inch Work, then you must set down your Thickness at one Brick-work; but If they be wrought nine Inches thick, (as sometimes they are when they stand high and alone

of MENSURATION. 93

alone above the Roof) then you must account your Thickness one and an half Brick, in Consideration of Wyth and Pargetting, and Trouble of Scaffolding.

It is customary in most Places to allow double Measure for Chimnies.

More EXAMPLES to exercise the foregoing PROPOSITIONS.

N. B. THE Areas of Parallelograms and Triangles being divided by one of their Dimensions, will give the other Dimension.

QUESTION XVII.

WHAT Difference is there between a Floor 48 Feet long, and 30 Feet broad; and two other, each of half the Dimensions?

$48 \times 30 = 1440$; and $24 \times 15 \times 2 = 720$; but 720 is half of 1440.

THEREFORE any plain Figure, whose Dimensions for Measuring are double to the Dimensions of another like Figure, contains four times the Area.

QUESTION XVIII.

A Mahogany Plank is 26 Inches broad; and I want a Yard and an half in Area saw'd off; what Distance from the End must the Line be struck?

$1\frac{1}{2}$ Yards Area = $13\frac{1}{2}$ Feet Area; and 26 Inches = 2,16 Feet.

THEN

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THEN $\frac{1350}{2,16} = 6,23$ Feet, the Distance from the End the Line must be struck.

QUESTION XIX.

A JOIST is $8\frac{1}{2}$ Inches deep, and $3\frac{1}{2}$ broad; I want a Scantling just as big again, that shall be $4\frac{1}{2}$ Inches broad; what will be the other Dimension?

$8,5 \times 3,5 \times 2 = 59,5$ and $\frac{59,5}{4,75} = 12,52$ Inches deep, the Answer.

QUESTION XX.

I HAVE a Girder 19 Inches by 13; but one that has but a Quarter of the Timber in it, so it be 10 Inches deep will serve my Purpose; how broad must it be?

$19 \times 13 = 247$, and $\frac{247}{4} = 61,75$. Then $\frac{61,75}{10} = 6,175$ Inches, the Breadth.

QUESTION XXI.

I HAVE a Roof 24 Feet, 8 Inches, by 14 6 Inches on the Flat; and I would have it covered with Lead at 8 lb to the Foot; what will it come to at 18 s. per Cwt.

24,8

of MENSURATION. 95

$24,5 \times 14,5 = 357,5$ F. the Area, and 8 lb.
 $= ,07232$ Cwt.; as 1 F. : $,07232$ Cwt. : : $357,5$ F.
 : $25,866553$ Cwt. the Weight.

THEN as 1 Cwt. : $,9$ l. : : $25,866453$ Cwt.
 : $23,2798$ l. &c. = 23 l. 5 s. 7 d. the Cost.

QUESTION XXII.

SUPPOSE a Mason and a Plumber should agree
 as follows:

THE Plumber to make the Mason a leaden
 Cistern, every Foot Square whereof should weigh
 18 lb. at 19 s. *per Cwt.* and the Dimensions to
 be 81 Inches long, 46 Inches deep, and 36
 Inches broad; with three Stays across it, of the
 same Strength, and each 18 Inches deep: And the
 Mason to pave with Purbeck Stone, at $7\frac{1}{2}$ *per Foot*,
 a Square, that should just balance the Cost of the
 Cistern; what must the Side thereof be?

$81 + 36 = 117$, which \times by $2 = 234 =$ the
 Length of the two Sides and two Ends; or the
 Girth of the Cistern; then $234 \times 46 = 10764$
 Inches, the Area of the Sides and Ends: And 81
 $\times 36 = 2916$ Inches, the Area of the Bottom.

Again, 36 , the Length of one Stay, \times by 18
 the Depth = 648 Inches, which $\times 3 = 1944$
 Inches, the Area of the three Stays. Then 10764
 $+ 2916 + 1944 = 15624$ Inches, the Area of
 the whole Cistern and Stays.

Note, 19 lb = 1696 Cwt. &c.

THEN, as 144 sq. In. : $,1696$ Cwt. : : 15624 sq.
 In. : $18,4016$ Cwt. The shortest Way to work
 this Proportion is, divide the third Term by the
 first Term, gives $108,5$, and \times the second Term
 by

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by this Quote, gives the fourth Term, which is the Weight of the Cistern.

THEN if 1 Cwt. : 95 l. :: 18,4016 Cwt. 17,48152 l. = 17 l. 9 s. 7½ the Expence that the Cistern does amount to.

THEREFORE, as, 03125 l. : 1 Ft. :: 17,48152 l. : 559,40864 Ft. the Area of the Square ; and by extracting the Square Root, you will have 23,65 Ft. &c. the Side thereof.

QUESTION XXIII.

I HAVE a Fleight of Iron Railing 42 Feet long, the Barrs whereof are ¾ of an Inch square, and the whole Weight is 12¾ Cwt. But I have a mind to change them for some that are Inch and ¾ strong, and must give in Exchange 4½ d. per lb; what will the whole come to?

IN order to solve this Question, say by the double Rule of Three:

IF 42 Feet of ¾ Inches Strength weigh 12¾ Cwt. what will 42 of 1¾ weigh ?

Or, in Decimals.

| F. | l. | Cwt. |
|----|------|-------|
| 42 | 5,75 | 12,75 |
| 42 | 1,8 | ? |

BUT the Strength of the Barrs are expressed by the Areas of the Ends ; therefore instead of the Sides of the Barrs, put the Squares, or Areas at the Ends, and then it will be,

| | | |
|----|-------|-------|
| 42 | 5,625 | 12,75 |
| 42 | 3,24 | |

Here the Blank falls under the third Place.

AND

of MENSURATION. 97

AND thus you should proceed, supposing there were two different Lengths of Railing concern'd; but as the Length of the new is to be the same with the old, therefore this is solvable by the single Rule, thus:

$$\begin{array}{rcl} \text{Strength. Cwt.} & \text{Strength. Cwt.} & \\ \text{As, } 5625 : 11,75 :: 3,24 : 73,44 \end{array}$$

$$\text{B d. B l. s.}$$

$$\text{IF } 1 : 4\frac{1}{2} :: 112 : 2 - 2$$

THEREFORE,

$$\begin{array}{rcl} \text{Cwt. l.} & \text{Cwt. l.} & \text{l. s. d.} \\ \text{As } 1 : 2,1 :: 73,44 : 154,224 = 154 - 4 - 5\frac{1}{2} \end{array}$$

QUESTION XXIV.

A TRIANGULAR Field that is 1777,7 Links in the Base, and 900 Links in the Perpendicular, brings in 36 l. per Annum; how much is it let for per Acre?

2) 900 (450; then $1777,7 \times 450 = 800000$ sq. Links which divide by 100000 (the square Links in one Acre) gives 8 Acres; then 8) 36 l. (4,5 l. = 4 l. 10 s. the Rent per Acre.

I BELIEVE the foregoing Examples are sufficient to illustrate the Uses of the first and second Proportions; as also to make the Business of Decimals and Duodecimals very easy and applicable; in which I suppose the Reader so well acquainted by this time, that I shall not give the Solution of any of the succeeding Questions in the Manner I have done those at the Beginning; but only shew how the Question is to be wrought, and give the principal Results of the Process, which I shall do Decimally, as being (in my Opinion) the most natural Way.

K

SECT:

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SECTION VII.

Of the Areas of divers Right-lined Figures.

PROPOSITION III.

The three Sides of a Triangle given to find the Area.

RULE.

From half the sum of the three Sides, subtract each side severally; then multiply the said half Sum, with the three Differences continually; and out of this Product extract the square Root; which Root will be the required Area.

QUESTION XXV.

A FIELD of a triangular Form hath it's Sides, 15, 14, and 13 Perches; what is the Area in square Perches?

$15 + 14 + 13 = 42$; and $\frac{42}{2} = 21$, the half Sum of the three Sides, or $\frac{1}{2}$ Perimeter.

$21 - 15 = 6$, the first Difference.

$21 - 14 = 7$, the second Difference.

$21 - 13 = 8$, the third Difference.

THEN $21 \times 8 \times 7 \times 6 = 7056$, whose square Root is 84 square Perches, the required Area.

QUEST

of MENSURATION. 99

QUESTION XXVI.

A FIELD of a triangular Form whose Sides are 380, 420 and 765 Yards, lets for 55 s. per Acre; how much does the whole bring in per Annum?

$$380 + 420 + 765 = 1565, \text{ and } \frac{1565}{2} = 782,5$$

Yards, the half Sum of the three Sides.

$$782,5 - 380 = 402,5, \text{ the first Difference.}$$

$$782,5 - 420 = 362,5, \text{ the second Difference.}$$

$$782,5 - 765 = 17,5, \text{ the third Difference.}$$

$782,5 \times 402,5 \times 362,5 \times 17,5 = 1998003710,9375$, whose square Root is 44699,034 square Yards, &c. which divided by 4840 (the square Yards in one Acre) gives 9,2374 Acres.

THEN, as 1 Acr. : 2,75 l. :: 9,2374 Acr. : 25,40285 l. = 25 l. 8 s. 0½ d. the Rent of the whole per Annum.

PROPOSITION IV.

Two Sides of a right angled Triangle being given, to find the other Sides.

THIS Proposition admits of two Cases.

First CASE.

THE two Sides (or Legs) perpendicular to each other being given, to find the other Side, called the Hypotenuse. Fig. 25.

K 2

RULE

R U L E.

SQUARE each Side, and add the Squares together, and out of their Sum extract the square Root; and it gives the Hypothenuſe as required.

Second C A S E.

If the Hypothenuſe and one of the Legs be given, to find the other Leg.

R U L E.

FROM the Square of the Hypothenuſe ſubtract the Square of the given Leg; and extract the ſquare Root out of the Remainder, gives the Leg required.

Q U E S T I O N XXVII.

I WANT the Length of a Shoar, that ſtrutting 14 Feet from the upright of a Building, may ſupport a Jaumb of 20½ Feet from the Ground?

$20,5 \times 20,5 = 420,25$, and $14 \times 14 = 196$, then $420,25 + 196 = 616,25$, whoſe ſquare Root is 24,82 Feet, &c. the Length of the Shoar required.

Q U E S T I O N XXVIII.

A LINE of 380 Feet will reach from the Top of a Precipice that ſtands cloſe by a Brook-side, to the

of MENSURATION. 101

the opposite Bank : Now it is known, the Precipice is 128 Feet high, how broad must the Brook be ?

$380 \times 380 = 144400$, and $128 \times 128 = 16384$, then $144400 - 16384 = 128016$, whose square Root is 357,79 Feet, &c. the Breadth of the Brook required.

QUESTION XXIX.

A LADDER $52\frac{1}{2}$ Feet long may be so placed in a Street, that it shall reach a Window 29 Feet from the Ground on one Side ; and by turning the Ladder over without removing the Foot, it will touch a moulding 40 Feet from the Ground on the other Side ; how broad is the Street ?

First, $52,5 \times 52,5 = 2756,25$, and $29 \times 29 = 841$, then $2756,25 - 841 = 1915,25$, whose square Root is 43,76 Feet, the Breadth between the Ladder and Building the first Situation.

Secondly, $40 \times 40 = 1600$, and $2756,25 - 1600 = 1156,25$, whose square Root is 34 Feet, the Breadth between the Ladder and Building, the second Situation.

THEN $43,76 + 34 = 77,76$ Feet, the Breadth of the Street required.

PROPOSITION V.

To find the Area of a Parallelogram not rectangular.

RULE.

FROM either of the greater Angles let fall a Perpendicular to the Side opposite that Angle ;

K 3

and

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and multiply that Perpendicular by the Length of the Side it falls on, and it gives the Area. *Fig. 26.*

QUESTION XXX.

THERE is a piece of Meadow Ground in Form of a Rhomboides, whose longest Side is 728 Yards, and the shortest Distance between the longest Sides [that is, the Perpendicular] is 358 Yards; Now supposing a Man can mow down the Grass standing on a square Rod in a Minute; how long will he be mowing the aforesaid Meadow, supposing him to work 14 Hours *per Day*.

$$\begin{aligned} 728 \times 358 &= 260624 \text{ Yds.} = 8615,669 \text{ Rods.} \\ &= 8615,669 \text{ Min.} = 10 \text{ D. } 3 \text{ H. } 3 \text{ M.} \end{aligned}$$

PROPOSITION VI.

To find the Area of a Trapezium.

RULE.

DIVIDE it into two Triangles by drawing a Line to any two of the Angles that are opposite, which Line is called the Diagonal; and from each of the other Angles let fall a Perpendicular to the Diagonal.

THEN Multiply the Sum of the two Perpendiculars by half the Diagonal; or, the Diagonal by half the Sum of the Perpendiculars, and the Product will be the Area.

OR find the Area of each Triangle, and their Sum will be the Area of the Trapezium,

QUES.

QUESTION XXXI.

WHAT is the Area of a Trapezium, whose Diagonal is 34 Feet, 9 Inches, and the Sum of the Perpendiculars is 28 Feet, 6 Inches?

By Duodimimals.

By Decimals.

F. I.

F.

34 - 9 The Diagonal 34.75

14 - 3 $\frac{1}{2}$ Sum of the Perpendicular. 14.25

8 - 8 - 3 17375

10 - 6 6950

476 - 0 48650

495 - 2 - 3 The Area 495.1875

QUESTION XXXII.

THERE is a Meadow in Form of a Trapezium, in which the Perpendiculars cannot be easily found; but the Diagonal is 538 Yards; which dividing it into two Triangles is a Side in each, in one Triangle the other Sides are 283 and 471, and in the other, the remaining Sides are 432 and 216; what is the Area?

$$538 + 471 + 283 = 1292, \text{ and } \frac{1292}{2} = 646.$$

THEN $646 - 538 = 108$, first Difference.

$646 - 471 = 175$, second Difference.

$646 - 283 = 363$, third Difference.

AND $646 \times 108 \times 175 \times 363 = 4432012200$, whose Root is 66573.35 Yards.

AGAIN,

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AGAIN, $538 + 432 + 216 = 1186$, and
 $\frac{1186}{2} = 593$.

THEN $593 - 538 = 55$, first Distance.

$593 - 432 = 161$, second Distance.

$593 - 216 = 377$, third Difference.

AND $593 \times 55 \times 161 \times 377 = 1979632655$,
 whose Root is 44493 Yards.

THEN, $66573,35 + 44493 = 111066,35$ sq.yds.
 $= 22$ Acr. 3 Rds. 31 sq. Pol. the ANSWER.

PROPOSITION VII.

To find the Area of a regular Polygon.

First, FIND the Center of the Figure, which is done thus:

IF the Figure has an even Number of Sides, then from any one Angle draw a Line to the Angle directly opposite, and the Middle of this Line is the Centre required. *Fig. 27.*

IF the Figure hath an odd Number of Sides; then bisect any two of the Sides, and from the Points of bisection draw Lines to the Angles directly opposite thereto, and the Point where these two Lines cut each other is the Centre of the Figure, as required. *Fig. 28.*

THE Center being thus found, from it let fall a Perpendicular to any one of the Sides, and multiply this Perpendicular by half the Sum of the Sides, and the Product is the Area.

QUES-

QUESTION XXXIII.

WHAT is the Area of a regular Pentagon whose Side is 25 Yards, and the Perpendicular let fall from the Center to one of the Sides is 17,2 Yards?

$25 \times 5 = 125$, and $\frac{125}{2} = 62,5$ the half Sum of the Sides.

THEN, $62,5 \times 17,2 = 1075$ square Yards, the Area.

BUT the Areas of regular Polygons is more expeditiously obtained by the Help of a Table constructed in the following Manner.

THE Side of any of the regular Polygons is supposed Unity or 1; then by a Trigonometrical Calculation, the Perpendicular is found, and from thence the Area by the foregoing Rule.

THUS having found the Area of any regular Polygon, whose Side is 1; the Area of another like Polygon, whose Side only is given may be found, without seeking after the Perpendicular; for as the Square of the Side of one Polygon is to it's Area, so is the Square of the Side of any other like Polygon, to it's Area.

BUT because the following Table shews the Area of Polygons from 3 Sides to 12, and the Side in each being Unity or 1, whose Square is also but 1; therefore multiply the Square of the Side of any given Polygon, by the corresponding Tabular Number, and it gives the Area.

Number

| <i>Number of Sides.</i> | <i>Names.</i> | <i>Multipliers.</i> |
|-------------------------|-------------------|---------------------|
| 3 | <i>Trigon</i> | ,4 3 3 0 1 3 |
| 4 | <i>Square</i> | 1,0 0 0 0 0 0 |
| 5 | <i>Pentagon</i> | 1,7 2 0 4 7 7 |
| 6 | <i>Hexagon</i> | 2,5 9 8 0 7 6 |
| 7 | <i>Heptagon</i> | 3,6 3 3 9 5 9 |
| 8 | <i>Octagon</i> | 4,8 2 8 4 2 7 |
| 9 | <i>Nonagon</i> | 6,1 8 1 8 2 7 |
| 10 | <i>Decagon</i> | 7,6 9 4 2 0 9 |
| 11 | <i>Undecagon</i> | 8,5 1 4 2 5 0 |
| 12 | <i>Duodecagon</i> | 9,3 3 0 1 2 5 |

E X A M P L E S.

WHAT is the Area of a regular Heptagon, whose Side is 12 Inches?

LOOK in the Table against the Name *Heptagon*, and under Multipliers you will find 3,633959, which multiplied by the Square of the given Side 12 produces 523,290096 the Area of a regular Heptagon whose Side is 12.

AFTER the same Manner may the Area of any other regular Polygon (whose Name is in this Table) be found.

PROPOSITION VIII.

To find the Area of irregular Figures.

R U L E.

DIVIDE the Figure into as many Triangles as may be, by drawing Lines from any one Angle to all the other Angles; and the Sum of the Areas of the several Triangles will be the Area of the Figure. *Fig. 29.*

BUT the Lines (frequently) may be better disposed, than by drawing them all from the same Angle, viz. by dividing the Figure into Trapezia, as many as may be, and leaving as few single Triangles as possible; for the Area of a Trapezium is more expeditiously obtained, than the Areas of the two Triangles which compose the Trapezium, found separately. *Fig. 30.*

Note. ALL right lined Figures may be divided into as many Triangles (without any of the dividing Lines cutting each other) as the Figures hath Sides, abating two.

QUESTION XXXIV.

SUPPOSE a Meadow an irregular Figure of 8 Sides, which is let for 34 Shillings *per Acre*; upon the Mensuration is divided into three Trapezia: In the first, the Diagonal is 4 Chains and 24 Links, and the Sum of the Perpendiculars are 3 Chains 67 Links; in the second the Diagonal is 7 Chains

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7 Chains 43 Links, and the Sum of the Perpendiculars are 5 Chains 38 Links; in the third the Diagonal is 6 Chains 78 Links, and the Sum of the Perpendiculars are 4 Chains 84 Links, what will the whole bring in *per Annum*? *Fig. 30.*

4 C. 24 L. = 424 L. and 3 C. 67 L. = 367 L.
then $367 \times \frac{1}{2} 424 = 77804$ L. the Area of the first Trapezium.

7 C. 43 L. = 743 L. and 5 C. 38 L. = 538 L.
then $743 \times \frac{538}{2} = 199867$ L. the Area of the Second.

AGAIN, 6 C. 78 L. = 678 L. and 4 C. 84 L. = 484 L, then $678 \times \frac{484}{2} = 164076$ L. the Area of the third.

THEN $77804 + 199867 + 164076 = 441747$ L. = 4,41747 Acres, which at 34 s. = 1,7 l. comes to 7,509699 l. = 7 l. 10 s. $2\frac{1}{2}$ d. the Answer.

SECTION VIII.

Of a CIRCLE and it's PARTS.

HAVING thus far treated of right lined Figures, I come next to the Propositions relating to a circle, of which there are many; but I shall content myself with mentioning only those of which there are frequent Use in practical Measuring;

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Measuring; in order to which, it will be necessary to say something concerning the Proportions made Use of hereafter about the Circle.

THE antient Mathematicians (and indeed some of the Moderns) made several Attempts to find Numbers that would accurately give the Proportion between the Diameter of a Circle and it's Circumference; but it was found impossible, and then concluded, it was to be done no other ways but by Approximation, that is, by Numbers whose Value continually approached nearer the Truth, without ever arriving exactly at it.

ARCHIMEDES found out a Proportion in whole Numbers, viz. of 7 to 22, that is, supposing the Diameter of a circle to be 7; he found the Circumference of the same Circle to be 22 nearly; for 22 is too much.

METIUS found out a Proportion of 113 to 355; that is, supposing the Diameter of a Circle to be 113, he found the Circumference of the same Circle 355 nearly; but it is also too great, tho' much nearer the Truth than *Archimedes*.

VAN CULEN made a greater Progress in this Affair than any before him; for he supposing the Diameter of a Circle to be 1, found (with prodigious Labour and Trouble) the Circumference be 3.141592653.5897932384.6264338327.950288 which was then thought so great a Work, and so curious a Performance, that the Numbers were cut on his Tomb-Stone in the Church-Yard of St Peter's at *Leyden*.

BUT by Methods of which the Moderns are now possessed, the same thing (and many others of the like intricate Nature) may be perform'd with abundantly less Labour and Trouble; as is sufficiently shewn by Mr *Jones* in his *Synopsis Palmariorum Matheseos*,

theseos, where he gives Mr *Machin's* Proportion of the Diameter to the Circumference true to 100 Places; that is, supposing (as *Van Culen* did) the Diameter to be 1, he found the Circumference to be 3,1415922653.5897932384.6264338327.9502884197.1693993751.0582097494.4592307816.4062862089.9862803482.5342117067.9 + true to 100 Places of Figures, and that with very little Time and Trouble; a notable Instance of the Superiority of the Methods used by the Moderns, over those of the Antients.

VAN CULEN's Proportion being of all others the most simple, as in some Cases saving the Trouble of Multiplication, and in others of Division, I shall all along confine myself to it; and here observe, that it is thought accurate enough to use no more of those Numbers than 3,1416; and thus increasing the fourth Place in the Decimals by Unity, the Deficiency in Operation arising by omitting, so many Figures [as must be done, to make the Work any thing tolerably easy] are in a Manner supply'd.

It may be expected that I should here shew how these Proportions were found; but as I write this chiefly for the Use of those who may not have had the Opportunity or Inclination to learn either Geometry or Algebra, I forbear it, knowing that if I should insert it, they would not readily understand it; and those who have spent some Time in the Study of Geometry and Algebra, doubtless have found it well done already in many Authors; and I think in none better than in the above-mention'd Book.

I SHALL now lay down some Theorems concerning Circles, which are already demonstrated in *Archimedes*, *Euclid*, and others.

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I. THE Area of every Circle is equal to the Area of a rectangled Triangle, one of whose Legs is a Line equal to the Radius, and the other Leg equal to the Circumference; and hence arises one Method for finding the Area of a Circle, viz.

MULTIPLY half the Circumference by half the Diameter, and the Product is the Area.

II. THE Diameters and Circumferences of all Circles are in a direct Proportion to each other, and so are their like Chords and Arches, as are also the Sides of their inscribed and circumscribed Figures; that is, as the Diameter of one Circle is to it's Circumference; so is the Diameter of any other Circle, to it's Circumference, &c.

III. THE Areas of all Circles are in the Proportion to each other, as are the Squares of their Diameters; or as the Squares of their Circumferences; and also as the Squares of the Sides of the inscribed and circumscribed Figures; that is, as the Square of the Diameter of one Circle is to it's Area; so is the Square of the Diameter of any other Circle to it's Area; and as the Square of one Circumference is to it's Area, so is the Square of any other Circumference to it's Area, &c.

FROM these three Theorems are deduced all the Proportions concerning a Circle used in practical Measuring.

THE following Table shews the useful Multipliers and Divisors calculated to shorten the Operations relating to Circles:

| | | | | |
|---------------------------------------|--|--|--|--|
| If the Dia- meter be 1, | The Circumfe- rence will be 3,1416. | The Area 7854. | The Side of a Square equal 8862. | The Side of an in- scribed Square 7071. |
| If the Circumfe- rence be 1, | The Diameter will be 3,1831. | The Area 207958. | The Side of a Square equal 2821. | The Side of the in- scribed Square 2251. |
| If the Side of a Square be 1, | The Diameter of the circumscribing Circle will be 1,4142. | Diameter of a Circle equal 1,1284. | Circumference of the Circum- scribing Circle 4,443. | Circumference of the Circle equal 3,545. |
| If the Area of a Circle be 1, | The Square of the Diameter will be 1,27324. | Square of the Circumference 12,5664. | The Area of the inscribed Square 6366. | |

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THIS Table will be sufficiently explained from the four following Propositions :

PROPOSITION IX.

The Diameter and Circumference of a Circle Being given to find the Area, and the Side of a Square whose Area shall be equal to that of the Circle or Square equal, and the Side of the Square inscribed in the Circle.

EXAMPLE

THE Diameter 1 and the Circumference 3,1416, what are the Rest ?

First, $\frac{3,1416}{2} \times \frac{1}{2} = 1,5708 \times ,5 = ,7854$
the Area per THEOR. I.

Secondly, THE square Root of ,7854 is ,8862 the Side of the square Equal per THEOR. III.

AND because the Side of the inscribed Square is the Hypotenuse of a right angled (*fig. 31*) Triangle whereof the Radli are the Legs : Therefore, the Radius $,5 \times ,5 = ,25$, which $\times 2 = ,5$, whose square Root is ,7071 the Side of the inscribed Square, per THEOR. III.

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PROPOSITION X.

The Circumference of a Circle being given to find first, the Diameter; secondly, the Area; thirdly, the Side of a Square Equal; and fourthly, the Side of the inscribed Square.

EXAMPLE.

THE Circumference 1, what are the Rest?

First, $3,1416 : 1 :: 1 : ,318309$ the Diameter, per THEOR. II.

Secondly, $\frac{,318309}{2} \times \frac{1}{2} = ,1591545 \times ,5 = ,07957725$, but ,07958 will do, which is the Area, per THEOR. I.

Thirdly, THE square Root of ,07958 is ,282 the Side of the square Equal per THEOR. III.

Fourthly, $\frac{,318309}{2} = ,1591545$, then ,1591545 $\times ,1591545 \times 2 = ,05068030974050$ whose square Root is ,225123, the Side of the inscribed Square, per THEOR. III.

THE Reason why the Square of the Radius is multiply'd by 2 is the same as in the last Example; viz. because the Side of the inscribed Square is the Hypotenuse of a right-angled Triangle, whereof the two Legs are each the Radii of the Circle. See PROSPT. IV. Fig. 31.

PROPOSITION XI.

The Side of a Square being given, to find the Diameter and Circumference of the circumscribing Circle, and also the Diameter and Circumference of the Circle equal.

EXAMPLE.

THE Side of a Square 1, what are the Rest?

First, the Diagonal of the Square being equal to the Diameter of the circumscribing Circle, (fig. 31) and it being also the Hypothenuſe of a right-angled Triangle, whereof each Side of the Square is a Leg; therefore per (PROP. IV.) $1 \times 1 \times 2 = 2$, whose square Root is 1,4142, the Diameter of the Circle circumscribing the Square, per THEOR. III.

Secondly, $1 : 3,1416 :: 1,4142 : 4,443$, the Circumference per THEOR. II.

Thirdly, $,7854 : 1 :: 1 : 1,27324$ and whose square Root is 1,1284 the Diameter of the Circle equal in Area to the Square, per THEOR. III.

Fourthly, $1 : 3,1416 :: 1,12837 : 3,544887192$ or 3,545 the Circumference of the Circle equal, per THEOR. II.

PROPOSITION XII.

The Area of a Circle being given ; to find the Squares of the Diameter and Circumference, and also the Side of the inscribed Square.

EXAMPLE.

THE Area 1, what are the Rest ?

First, $7854 : 1 :: 1 : 1,27324$, the Square of the Diameter, *per* THEOR. III. And by the last Example the Circumference of a Circle, whose Area is 1, was found to be 3,544887192, which squared gives 12,5662252, &c. the Square of the Circumference.

Secondly, In the last Example the Diameter of a Circle whose Area is 1, was found to be 1,12837, then $\frac{1,12837}{2} = ,56418$ and $,56418 \times ,56418 = ,636598$, &c. or ,6366 the Area of the inscribed Square.

FOR the Square of half the Radius is (*per* PROPOS. IV.) equal to twice the Square of half the Side of the inscribed Square and every Square being equal to four times the Square of half the Side ; therefore twice the Square of the Radius, is equal to the inscribed Square, *fig.* 32.

THE following Examples will shew the Application of the Numbers in the Table found in the four foregoing Propositions.

EXAMPLE

EXAMPLE I.

SUPPOSE a Circle whose Diameter is 12 Inches; what is the Diameter, Area, Side of the Square Equal, and Side of the inscribed Square?

Dia. Circumf. Dia. Circumf. } The Circumference.
 $1 : 31416 :: 12 : 37,6992$

sq.D. Area. sq.D. Area. } The Area.
 $1 : ,7854 :: 144 : 113,0976$

Dia. S.S.E. Dia. S.S.E. } The Side of the Square Equal.
 $1 : ,8862 :: 12 : 10,6344$

Dia. S.I.S. Dia. S.I.S. } The Side of the inscrib'd Square.
 $1 : ,7071 :: 12 : 8,4852$

EXAMPLE II.

SUPPOSE the Circumference 12, required the same things as before?

C. D. C. D. } The Diameter.
 $1 : ,31831 :: 12 : 3,81972$

S.C. A. S.C. A. } The Area.
 $1 : ,07958 :: 144 : 11,45952$

C. S.S.E. C. S.S.E. } The Side of the Square Equal.
 $1 : ,2821 :: 12 : 3,3852$

C. S.I.S. C. S.I.S. } The Side of the inscrib'd Square.
 $1 : ,2251 :: 12 : 2,7012$

EXAM

EXAMPLE III.

LET the Side of a Square be 12, what are the Diameter and Circumferences of the circumscribing Circle, and Circle Equal?

SS. D.C.C. SS. D.C.C. } The Diameter of the
 $1 : 1,4142 :: 12 : 16,9704$ } circumscribing Circle.

SS. C.C.C. SS. C.C.C. } The Circumference of the
 $1 : 4,443 :: 12 : 53,316$ } circumscribing Circle.

S.S. D.C.E. S.S. D.C.E. } The Diameter of the
 $1 : 1,128 :: 12 : 13,536$ } Circle Equal.

S.S. C.C.E. S.S. C.C.E. } The Circumference of
 $1 : 3,545 :: 12 : 42,54$ } the Circle Equal.

EXAMPLE IV.

LET the Area of a Circle be 12, what is the Diameter and Circumference thereof, and the Side of the inscribed Square?

A. S.D. A. S.D. } whose square Root is
 $1 : 1,27324 :: 12 : 15,27888$ } 3,9087 the Diameter.

A. Sq.Circ. A. Sq.Circ. } whose sq. Root
 $1 : 12,5664 :: 12 : 150,7946$ } is 12,2798 the
Circumference.

A. Sq.I. A. Sq.I. } whose square Root is 2,7639
 $1 : ,6366 :: 12 : 7,6392$ } the Side of the inscrib'd Square.

IF the Diameter or Circumference, &c. was any other Number, the Method of Operation would be exactly the same.

PROPO-

PROPOSITION XIII.

To find the Area of a Semicircle or Quadrant.

R U L E.

MULTIPLY half the Arch Line by the Radius, and it givis the Area.

E X A M P L E I.

WHAT is the Area of a Semicircle, whose Diameter is 20 ?

As, 1 : 3,1416 :: 20 : 62,832, the whole Circumference ; the half is 31,416, the Arch Lines. Then $\frac{31,416}{2} \times \frac{20}{2} = 157,08 \times$

10 gives 157,08 the Area of the Semicircle.

E X A M P L E II.

SUPPOSE the Diameter of a Circle be 16, what is the Area of a Quadrant (or fourth Part) thereof ?

As 1 : 3,1416 :: 16 : 50,2656, whole the Circumference ; the fourth part is 12,5664 the Length of the Arch-Line of the Quadrant. Then $\frac{12,5664}{2}$

$\times \frac{16}{2} = 6,2832 \times 8 = 50,2656$, the Quadrant's Area.

THE most expeditious Method is thus :

First, For a Semicircle multiply 3,1416 by a fourth of the given Diameter, and multiply this

this Product by half the given Diameter, and it gives the Area.

Secondly, For a Quadrant multiply 3,1416 by an eighth Part of the given Diameter, and this Product by half the given Diameter, and it gives the Area.

THE Truth of these two Rules is manifest from the foregoing Operations.

PROPOSITION XIV.

Having the Diameter of a Circle, and the Distance from the Center a Chord-Line is to be drawn, to find the Length of that Chord.

R U L E.

FROM the Radius subtract the Distance from the Center the Chord-Line is to be drawn, and the Remainder is the versed Sine; then multiply the Difference between the Diameter and the versed Sine, by the versed Sine, and out of the Product extract the square Root, gives the Length of half the Chord, which doubled gives the Chord as required. *Fig. 33.*

E X A M P L E.

SUPPOSE a Circle 21,2 in Diameter; what is the Length of the Chord-Line drawn 5,6 from the Center?

$\frac{21,2}{2} = 10,6$ the Radius; then $10,6 - 5,6 = 5$, the versed Sine, and $21,2 - 5 = 16,2$, the Difference; then $16,2 \times 5 = 81$, whose square Root

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Root is 9 = half the Chord: Therefore the whole Chord is 18.

HENCE it will be easy to find the Diameter by having the Chord and versed Sine given; for divide the Square of half the Chord by the versed Sine, and to the Quotient add the versed Sine gives the Diameter: For Example; suppose the Chord 18, and the versed Sine 5, what is the Diameter?

$\frac{18}{2} = 9$, the half Chord, and $9 \times 9 = 81$;
then $\frac{81}{5} = 16,2 + 5 = 21,2$, the Diameter sought.

IT will be very easy to find the Chord of half the Arch, for it will be the Hypothenuse of a right angled Triangle, whereof the versed Sine will be one Leg, and the half Chord the other Leg; wherefore extracting the square Root out of the Sum of the Squares of the versed Sine and half Chord, (per PROP. IV.) will give the Chord of half the Arch. Fig. 33.

PROPOSITION XV.

To find the Length of any Arch of a Circle, by having the Chord of that Arch and the Diameter of the Circle given.

RULE.

DIVIDE the Cube of the Chord by six times the Square of the Diameter, and to the Quotient add the Chord, gives the Length of the Area, very near. But if you would have it more accurate, then divide three times the fifth Power of the Chord, by 40 times the fourth Power of the Diameter, and

M

this

this Quotient add to the former Sum, gives the Length of the Arch very exact.

THERE are two other Rules given in the Books of Mensuration, viz.

MULTIPLY the Chord of half the Segment by 8, and from the Product subtract the Chord of the whole Segment; this Remainder divide by 3, gives the Arch-Line.

OR thus: To the double Chord of half the Segment's Arch, add one third of the Difference between the Chord of the whole Segment, and the double Chord of half the Segment's Arch, gives the Length of the Arch-Line.

THE Arch is obtained by either of these last Rules very easily; supposing the Chord of half the Arch be given; but if that be first to be sought, from the Chord and Diameter, it requires a tedious Operation.

EXAMPLE.

SUPPOSE the Diameter 40, and the Chord 30; what is the Length of the Arch?

THIS Example I shall work by each Rule.

First, $30 \times 30 \times 30 = 27000$, the Cube of the Chord, $40 \times 40 \times 6 = 10600$; then $\frac{27000}{10600} = 2,547, \text{ \&c.}$ and $30 + 2,547 = 32,547$, the Length of the Arch: But if more Exactness be required, then $30 \times 30 \times 30 \times 30 \times 30 \times 30 = 72900000$, and $40 \times 40 \times 40 \times 40 \times 40 = 102400000$; then $\frac{72900000}{102400000} = ,711$, and $32,547 + ,711 = 33,258$, the Length very near.

Secondly, By the other Methods; but first to find the Chord of half the Arch.

If

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If a Line be drawn from the Center to one End of the Chord, then it is plain, that if from the Square of that Radius be taken the Square of half the Chord, the square Root of the Remainder will be that Part of the Diameter intercepted between the Center and the Chord, which subtracted from the Radius leaves the versed Sine; then the Square of the versed Sine, and the Square of the Semi-chord added together, the square Root of their Sum is the Length of the Chord of half the Arch.

See the Operation. $\frac{40}{2} = 20$, the Radius, 20

$\times 20 = 400$, and $\frac{30}{2} = 15$, the Semi-chord, 15

$\times 15 = 225$; then $400 - 225 = 175$, whose square Root is 13,2287, and $20 - 13,2287 = 6,7713$, the versed Sine; then $6,7713 \times 6,7713 = 45,85050369$, and $225 + 45,85050369 = 270,85050369$, whose square Root is 16,4575, the Chord of half the Arch; then *per Rule second*.

$16,4575 \times 8 = 131,66$, and $131,66 - 30 = 101,66$, then $\frac{101,66}{3} = 33,88$, the Arch's Length.

AGAIN, *per Rule third*, $16,4575 \times 2 = 32,915$, and $32,915 - 30 = 2,915$, and $\frac{2,915}{3} = ,9713$, then $32,915 + ,9713 = 33,8863$, the Length of the Arch.

If the Length of the Chord of half the Arch be taken mechanically, I think the second Rule the most expeditious, and is accurate enough for Practice, for the Difference of the Result in all the Methods is but small.

PROPOSITION XVI.

To find the Area of a Sector.

RULE.

FIND the Length of the Arch intercepted between the two Radii, then multiply half that Arch by the Radius, and the Product is the Area. *Fig. 34.*

EXAMPLE.

WHAT is the Area of a Sector, the Radius being 24,5 and the Length of the Arch 62,6?

$$\frac{62,6}{2} = 31,3; \text{ then } 31,3 \times 24,5 = 766,85,$$

the Area.

IF the Length of the Arch was greater than a Semicircle; then find the whole Circumference, and also the Length of the Arch wanting to complete the Circumference, which subtracted from the Circumference leaves the Length of the Arch in the given Sector, the Area of which find as before. *Fig. 35.*

PROPOSITION XVII.

To find the Area of a Segment of a Circle.

RULE.

FIRST find the Center of a Circle, from which draw a Radii to the Ends of the Chord, and find the Area of that Sector, from which subtract the Area of the Triangle contain'd under the Chord and the two Radii, and the Remainder is the Area of the Segment. *Fig. 36.*

EXAM-

EXAMPLE.

SUPPOSE a Segment of a Circle has a Chord-Line 24 Inches, and the versed Sine 9 Inches ; what is the Area?

$\frac{24}{2} = 12$, and $12 \times 12 = 144$, and $\frac{144}{9} = 16$, and $16 + 9 = 25$, the Diameter (*per PROPOS. XIV.*) then $\frac{25}{2} = 12,5$, the Radius. Again, the Semi-chord $12 \times 12 = 144$, and the versed Sine $9 \times 9 = 81$, then $144 + 81 = 225$, whose square Root is 15 the Chord of half the Segment (*per PROPOS. XV.*) then $15 \times 8 = 120$, and $120 - 24 = 96$, and $\frac{96}{3} = 32$, the Length of the Arch, half is 16 ; then 12,5 the Radius multiply'd by 16, gives 200 = the Area of the Sector.

AGAIN, 12,5 (the Radius) — 9 (the versed Sine) leaves 3,5 the Height of the Triangle made by the Chord and Radii, then $\frac{24}{2} = 12 \times 35 = 42$, the Area of that Triangle ; then $200 - 42 = 158$, the Area of the Segment.

THE same may be done without finding the Length of the Arch, by the following Rule.

MULTIPLY the Difference between the Radius and versed Sine by 8 times the Radius, and this Product subtract from 14 times the Square of the Radius, from this Difference subtract 6 times the Square of the Difference between the Radius and versed Sine ; and this last Remainder divide by 9 times the Radius, added to 6 times the Difference between the Radius and versed Sine, and the Quo-

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tient multiply by half the Length of the Chord, and the Product is the Area.

TAKE the foregoing Example in order to compare the Methods, viz.

THE Chord 24 and the versed Sine 9, what is the Area ?

$$\frac{24}{2} = 12 \text{ the } \frac{1}{2} \text{ Chord, and } 12 \times 12 = 144, \text{ and}$$

$$\frac{144}{9} = 16, \text{ and } 16 + 9 = 25 \text{ and } \frac{25}{2} = 12,5,$$

the Radius, and $12,5 - 9 = 3,5$, the Difference between the Radius and versed Sine.

$$12,5 \times 8 \times 3,5 = 350, \text{ and } 12,5 \times 12,5 \times 14 = 2187,5, \text{ then } 2187,5 - 350 = 1837,5, \text{ and } 3,5 \times 3,5 \times 6 = 73,5, \text{ then } 1837,5 - 735 = 1764.$$

$$\text{AGAIN, } 12,5 \times 9 = 112,5, \text{ and } 3,5 \times 6 = 21, \text{ then } 112,5 + 21 = 133,5, \text{ and } \frac{1764}{133,5} = 13,21348,$$

then $13,21348 \times 12 = 158,56176$, the Area ; nearly the same as by the former Rule.

PROPOSITION XVIII.

Having the Diameter of a Circle given, to find the Diameter of another Circle, whose Area shall increase or decrease by a given Ratio.

R U L E.

MULTIPLY the Square of the given Circle's Diameter by the intended Increase, and out of that Product extract the square Root, which Root will be the Diameter of the Circle required.

BUT if the required Circle is to be less than the given Circle, then instead of Multiplying, divide by the

of MENSURATION. 127

the intended Decrease, and out of the Quotient extract the square Root.

E X A M P L E.

How much must that Circle be in Diameter, whose Area is 12 times as much as a Circle of 25 Inches in Diameter?

$25 \times 25 = 625$, the square of the given Diameter, which multiply'd by 12, the intended Increase gives 7500, whose square Root is 86,6 the Diameter required; but had it been to be decreased 12 times, then $\frac{625}{12} = 52,0833$, whose square Root is 7,237 the Diameter of a Circle, whose Area is a twelfth Part of the Area of that Circle whose Diameter is 25.

PROPOSITION IV.

To find the Area of compound Figures.

FIGURES are said to be compound, when the Sides thereof consist of both right and curved Lines, and how to find the Area of such Figures; the Way is, to every Curve Side draw a Chord, and then the Figure [exclusive of the Segments contain'd under the Curve Line and Chord] will be reduced to a right-lined Figure, whose Area find as directed in PROP. VIII. to which, if the Area of the Segments (found as per PROP. XVII.) be added, the Sum will be the whole Area. Fig. 37.

EXAMPLE

EXAMPLE.

SUPPOSE a Close of five Sides, two whereof are the Arches of Circles ; now the Chords being drawn, the Figure will be a five-sided right-lined Figure, which being (by Lines drawn) divided into a Trapezium and a Triangle ; in the Trapezium the Diagonal is 7 Chains, and the Sum of the Perpendiculars $6\frac{1}{2}$ Chains ; in the Triangle the Base is 4 Chains, and the Perpendicular 6 Chains ; in one Segment the whole Chord is 6 Chains, and the Chord of half the Arch is $3\frac{1}{2}$ Chains ; in the other Segment the whole Chord is 4 Chains, and the Chord of half the Arch is $2\frac{1}{2}$ Chains ; what is the Area of this Figure ?

First, $\frac{7}{2} = 3,5$, then $3,5 \times 6,5 = 22,75$, the Area of the Trapezium ; *per* PROP. VI.

Secondly, $\frac{6}{2} = 3$, then $3 \times 4 = 12$, the Area of the Triangle ; *per* PROP. II.

Thirdly, In the first Segment the Cord of half the Arch is 3,5, and half the whole Chord is 3, then $3,5 \times 3,5 = 12,25$, and $3 \times 3 = 9$, and $12,25 - 9 = 3,25$, whose square Root is 1,8 the versed Sine, then $\frac{9}{1,8} = 5$, and $5 + 1,8 = 6,8$, and $\frac{6,8}{2} = 3,4$, the Radius, and $3,4 - 1,8 = 1,6$, the Difference between the versed Sine and Radius ; then $3,4 \times 8 \times 1,6 = 43,52$, and $3,4 \times 3,4 \times 14 = 161,84$, and $161,84 - 43,52 = 118,32$, and

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$1,6 \times 1,6 \times 6 = 15,36$, and $118,32 - 15,36$
 $= 102,96$, and $3,4 \times 9 = 30,6$, and $1,6 \times 6$
 $= 9,6$, and $30,6 + 9,6 = 40,2$, then $\frac{102,96}{40,2}$
 $= 2,56$, &c. the Area.

Fourthly, In the second Segment the Chord of
 half the Arch is 2,5 and half the whole Chord
 is 2, then $2,5 \times 2,5 = 6,25$, and $2 \times 2 = 4$,
 and $6,25 - 4 = 2,25$, whose square Root is 1,5
 the versed Sine, then $\frac{4}{1,5} = 2,6$, and $2,6 + 1,5$,
 $= 4,16$, and $\frac{4,16}{2} = 2,08$, the Radius, and $2,08$
 $- 1,5 = ,58$, the Difference between the Radius
 and versed Sine, then $2,08 \times 8 \times ,58 = 9,72$,
 and $2,08 \times 2,08 \times 14 = 60,7638$, and $60,7638$
 $- 9,72 = 51,0438$, and $,58 \times ,58 \times 6$
 $= 2,0418$, and $51,0438 - 2,0418 = 49$, and $2,08$
 $\times 9 = 18,75$, and $,58 \times 6 = 3,5$, and $18,75$
 $+ 3,5 = 22,25$, then $\frac{49}{22,25} = 2,2$ the Area, then
 $21,75 + 12 + 2,56 + 2,2 = 38,51$ square Chains,
 and 10 square Chains being 1 Acre, therefore the
 Area of the Glose is 3 Acres, 8 square Chains, 8
 square Poles, &c.

II. QUESTION SECT.

SECTION IX.

PRACTICAL QUESTIONS.

IF what has been already said be well consider'd and understood, the Business of superficial Measuring so far as a right Line and a Circle is concern'd, must needs be very easy and familiar, and therefore I think there needs no more be said on this Subject; but before I close this Part, I will insert a few Questions to exercise the young Measurer in the Business already done.

QUESTION I.

A ROUND Pillar 7 Inches over, is sufficient to carry a certain Weight, of what Diameter is the Column that contains 10 times the Stone on the same Length?

$7 \times 7 \times 10 = 490$, whose square Root is 22,135 Inches, the Diameter as required.

QUESTION II.

A BREWER hath a Cistern which is fill'd by three Pipes, each of 3 Inches bore in a certain Time, of what Diameter must the Bore of that Pipe

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Pipe be that in the same time will throw him in $2\frac{1}{2}$ times as much Water?

THE Quantity of Water thrown in being as the Squares of the Diameters, therefore $3 \times 3 \times 3 \times 2,5 = 67,5$, whose square Root is 8,215 Inches, &c. the Diameter sought.

QUESTION III.

SUPPOSE a Yard of Cable 9 Inches in Compass weighs 22 lb., what will a Fathom weigh of that 9 Inches in Diameter?

i Circumf. : 31831 *Diam.* :: 9 *Circumf.* : 2,86479 *Diam.* whose square is 8,2066, &c. then 8,2066 : 22 lb :: 81 (9×9) : 217,14, which \times by 2, gives 434,28, the Weight sought.

QUESTION IV.

I WANT in a Garden a Circular Pond, that shall just take up half an Acre, how long must the Chord be that will strike the Circle?

THE half Acre contains 2420 square Yards, therefore $1 : 1,2732 :: 2420 : 3081,2408$ square of the Diameter, whose square Root is 55,5 the Diameter, the half of which is 27,75 Yards, the Length of the Line sought.

QUESTION

QUESTION V.

AGREED for an Oaken Curb to a round Well, 8 *d.* per Foot square; the Breadth of the Curb is to be $7\frac{1}{2}$ Inches, and the Diameter of the Well within the Brick-Work is $3\frac{1}{2}$ Feet, what will be the Expence?

$3\frac{1}{2}$ F. = 42 In. then $42 \times 42 \times ,7854$
= 1385,4456, the Area within the Curb.

$42 + 7,25 + 7,25 = 56,5$, then $56,5 \times 56,5$
 $\times ,7854 = 2507,19315$ the Area of a Circle equal
the Area of the Well and Curb, then $2507,19315$
 $- 1385,4456 = 1121,74755$, the Area of the
Curb; now 8 *d.* = 0,8 *l.* therefore as $144 : ,08$
 $:: 1121,74755 : ,259671$ = 5 *s.* 2 *d.* the Expence
sought.

QUESTION VI.

I WOULD have in a Garden a circular Pond with a circular Island in the Middle thereof; the Diameter of the Pond must be 100 Yards, and the Circumference of the Island the same; what will the digging the Pond come to at 18 *d.* per Foot square on the Surface?

$100 \times 100 \times ,7854 = 7854$ Yards, the Area
of the Pond and Island; $100 \times 100 \times ,07958$
= 795,8, the Area of the Island; then 7854
 $- 795,8 = 7058,2$ Yards, the Area of the Pond
= 63523,8 Feet; then $1 : ,075$ *l.* :: $63523,8$
: 4764,285 = 4764 *l.* 5 *s.* 8 $\frac{1}{2}$ *d.*

QUESTION

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QUESTION VII.

SUPPOSE the Expence of paving a Semicircular Plot, at 2 s. 4 d. per Foot amounted to 10 l. what is the Diameter thereof?

2 s. 4 d. = ,116, and if ,116 l. 1 F. :: 10 l. $\$5,714\frac{1}{2}$ the Semicircle's Area, and $\$5,714\frac{1}{2} \times 2 = 11,428\frac{1}{2}$ the Circle's Area; then 1 : 1,2732 :: 11,428 $\frac{1}{2}$: 218,26285713, whose square Root is 14,7734 the Diameter sought.

QUESTION VIII.

SUPPOSE St James's Square to be 180 Yards long and 150 Yards broad, in which there is an Octagonal Gravel Walk, one of whose Sides is (suppose) 28 Yards; what did the Paving the rest with Purbeck Stone come to at 3 s. 6 d. per Yard?

180 \times 150 = 27000 Yards, the Area of the Square.

28 \times 28 \times 4,828427 = 3685,486768 Yds. the Area of the Octagon.

THEN 27000 Yds. — 3685,486768 = 23314,513232 Yds. what was paved; and 1 Y. : ,175 l. :: 23314,513232 Yds. : 4080,0398156 l. = 4080 l. 00 s. 9 $\frac{1}{2}$ d. the whole Cost.

N QUESTION

QUESTION IX.

WHAT is the Area of the Segment of a Circle whose Diameter is 50 Inches; supposing the Section made 14 Inches from the Center?

$\frac{50}{2} = 25$, the Radius, and $25 - 14 = 11$, the versed Sine. Then $25 \times 25 = 625$, and $14 \times 14 = 196$, and $625 - 196 = 429$, whose square Root is 20,7, &c. and $20,7 \times 2 = 41,4$, the Length of the Chord, *per PROP. XIV.*

AGAIN $25 \times 8 \times 14 = 2800$, and $25 \times 25 \times 14 = 8750$, and $8750 - 2800 = 5950$, and $14 \times 14 \times 6 = 1176$, and $5950 - 1176 = 4774$, the Dividend.

AGAIN, $25 \times 9 = 225$, and $14 \times 6 = 84$, and $225 + 84 = 309$, the Divisor.

THEN, $\frac{4774}{309} = 28,317$, and $28,317 \times 20,7 = 586,1619$, the Area sought, *per PROP. XVII.*

QUESTION X.

THREE Men bought a circular Cheese, 14 Inches in Diameter, which cost them 7 s. 6 d. whereof *A* pays 1 s. 4 d. *B* 2 s. 10 d. and *C* 3 s. 4 d. now they agree that it shall be divided from the Center to the Circumference; that is, it should be cut into three Sectors, the Area of which should bear the same Proportion to each other, as the Price each Man paid; what Part of the Circumference will fall to each Man's Share, together with the Areas?

$14 \times 14 = 196$, and $1 : 7854 :: 196 : 153,8384$ the whole Area.

7 s. 6 d.

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7s. 6d. = ,375 l. 1s. 4d. = ,06 l. 2s. 10d. = ,1416 l. 3s. 4d. = ,18 l.

,375 : 153,8384 :: ,06 : 27,349, *A's* Area.

,375 : 153,8384 :: ,1416 : 58,1167, &c. *B's* Share.

,375 : 153,8384 :: ,18 : 68,372, &c. *C's* Area.

As 1 : 3, 1416 :: 14 : 43,9824, the whole Circumference. Then 153,8384 : 43,9824 :: 27,349

7,819 Circumference of *A's*. 153,8384 : 43,9824

: 58,1167 : 16,620 Circumference of *B's*. 153,8384

43,9824 :: 68,372 : 19,547 Circumference of *C's*.

But the Circumference of each Man's Part may

more easily be found, thus: as ,375 : 43,9824

: ,06 : 7,819, *A's* Part of the Circumference,

,375 : 43,9824 :: ,1416 : 16,62, *B's* Part, ,375

43,9824 :: ,18 : 19,547, *C's* Part of the Circumf.

QUESTION XI.

A, B, and C bought a Grinding-Stone of 21 inches in Diameter, each paying a third Part; what Part of the Diameter must each grind down?

THIS Question is answer'd by reckoning each Man to grind away one third of the Area.

21 X 21 X ,7854 = 346,3614, the Area, and

$\frac{46,3614}{3} = 115,4538$, each Man's Area, which

taken from the whole, leaves 230,9076, the Area

of the Circle remaining, when one Man has ground

away his Share. Then 1 : 1,2732 :: 230,9076

293,99155632, whose square Root is 17,1461,

the Diameter of two Mens Shares next the Center.

again, 1 : 1,2732 :: 115,4538 : 146,99577816,

whose square Root is 12,1241, the Diameter of one

Man's Share next the Center. Then 21 — 17,1461

= 3,8539, the first Man's Part; and 17,1461

— 12,1241 = 5,022, the second Man's Part; and

the third 12,1241.

QUESTION

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ON a fair Country Green there once did stand,
A circ'lar Rail about a Piece of Land :
The greatest Part o' the Rail is now quite gone,
And the Ground-Plot defac'd it stood upon ;
But by what's left, I can find the Chord-Line,
And also get the Length o' the versed Sine ;
The Chord-Line's 40 Yards the other's 9.
Now I have order'd a new Rail be made,
Also a circ'lar Walk within be laid ;
Whose Breadth's twelve Feet ; the Price agreed is
known ;

For one Yards Length o' the Rail to be 1 Crown ;
And for one Square Yards of the Gravel Way,
I to the Workman Eighteen Pence must pay ;
Now tell to me the whole Expence I pray ?

$\frac{40}{2} = 20$, and $20 \times 20 = 400$, and $\frac{400}{9} = 44\frac{4}{9}$
and $44\frac{4}{9} + 9 = 53\frac{4}{9}$, the Diameter of the Rail
 $1 : 3,1416 :: 53\frac{4}{9} : 167,90106$, or $167,9$, the
Circumference, or Length of the Rail ; $\frac{53\frac{4}{9}}{2}$
 $\times \frac{167,9}{2}$, or $26,77 \times 83,95 = 2243,3305$, Area of
the outward Circle. 12 Feet = 4 Yards, and
 $4 \times 2 = 8$, the Walk's Breadth doubled. $53\frac{4}{9} - 8$
 $= 45\frac{4}{9}$, the Diameter of the inner Circle of the
Walk, and $45\frac{4}{9} \times 45\frac{4}{9} \times ,7854 = 1621,99713$,
the Area of the inner Circle, and $2243,3305$
 $- 1621,99713 = 621,333424$, the Walk's Area,
 $5 s. = ,25 l.$ then $167,9 \times ,25 l. = 41,974$
 $= 41 l. 19 s. 5\frac{1}{2} d.$ the Price of the Rail. $18 d.$
 $= ,075 l.$ then $621,333424 \times ,075 = 46,6000068$
 $= 46 l. 12 s.$ the Walk's Price ; therefore $41,974$
 $+ 46,6 = 88,574 l. = 88 l. 11 s. 5\frac{1}{2} d.$ the whole
Expence.

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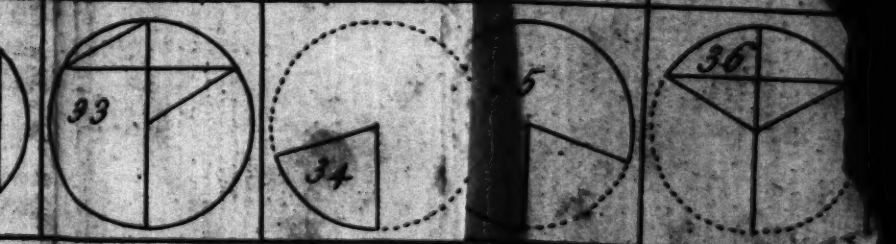
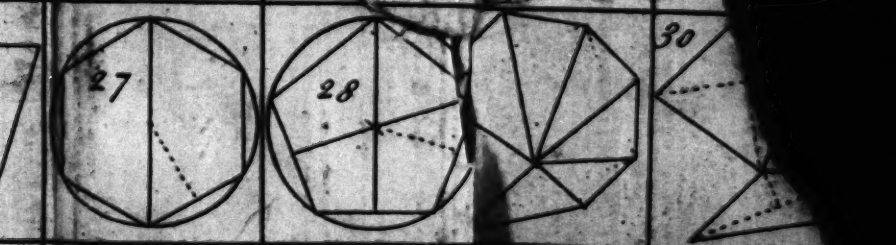
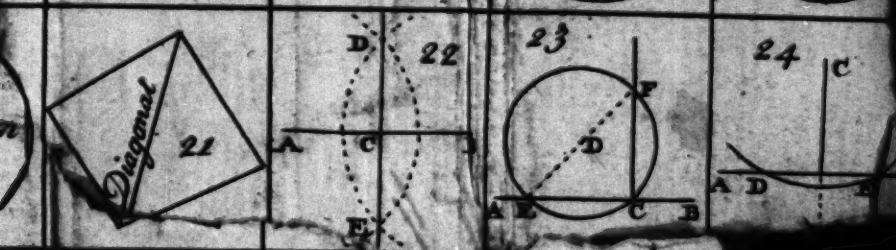
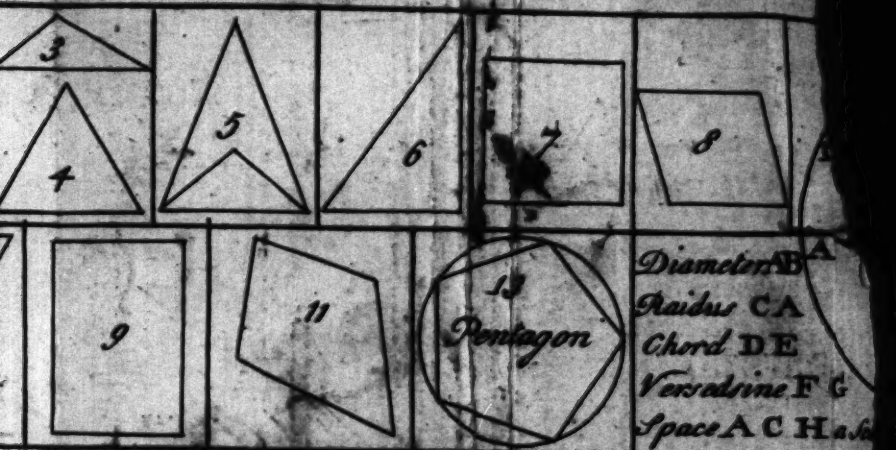
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Plate. 1.





OF
SOLID
MEASUREMENT.

PART II.

I SHALL *first* lay down the necessary Definitions; *secondly*, give the common Methods of measuring Timber; and *thirdly*, give the Methods whereby the Solidity of various Sorts of Figures are calculated.

SECTION I.

DEFINITIONS.

I. A SOLID is a Figure contained under three Dimensions, *viz.* Length, Breadth, and Depth or Thickness.

II. A PRISM is a solid, whose Sides are Parallel (or more properly the Lines bounding the Sides are Parallel to each other) as are also the Ends.

III. A CUBE is a Prism that hath equal Length, Breadth, and Depth [such a Figure as a Die] and is contain'd under six equal square Planes. *Plate 2.*

Fig. 1.

IV. A PARALLELOPIPEDON is a Figure that hath it's Sides bounded or included within four equal Parallelograms, and two square Bases or Ends. *Fig. 2.*

V. A CYLINDER [or Solid like the Rolling-Stone of a Garden] is only a round Prism, having the Bases or Ends perfect Circles. *Fig. 3.*

VI. A PYRAMID is a Solid, whose Base is a Polygon, and the Sides are Triangles, whose Tops all meet in a Point; which Point is called the Vertex of the Pyramid. *Fig. 4.*

VII. A CONE is a Solid standing on a circular Base, and tapereth away to a Point called it's Vertex, in such a Manner that all Lines from the said Vertex along the flant Surface are strait lines. *Fig. 5.*

VIII. A SPHERE or Globe is a Solid bounded within one regular Surface, and is formed by turning a Semicircle about it's Diameter. — The Diameter of a Sphere is called it's *Axis*, and is equal to the Diameter of the generating Semicircle. *Fig. 6.*

IX. If the small End of a Pyramid or Cone be cut off, the Remainder is called a *Fruustum*; and the Fruustum of a Sphere is any Slice cut from the whole Sphere; and if a Fruustum of a Pyramid or Cone be cut thro' diagonally, from the Extremitie of one Side at the lesser End, to the Extremitie of the Base on the other Side, each of these Pieces is called a *Haaf*, or an *Ungula*, that being the greatest which has the greatest Base to it.

SECTION II.

PROPOSITION I.

To measure round Timber by the common Method.

RULE.

TAKE the Length of the Tree in Feet and Inches; then with a small Cord or Chalk-line girt the Tree at an equal Distance from each End; one fourth Part of the Circumference is called by Artificers the *Girt*, and is (by them reckoned) the Side of a Square whose Area is equal to the Area of the Tree, supposing it to be cut through in the Place where it was girted; then multiply the Square of this Girt by the Length of the Tree, and it gives the Content.

WHAT is the solid Content of a Tree, whose Compass is 32 Inches, and the Length 9 Feet?

$$\frac{32}{4} = 8 \text{ Inches the Girt, and } 8 \text{ Inches} = \frac{2}{3} \text{ Foot.}$$

$$\frac{2}{3} \times \frac{2}{3} = \frac{4}{9}, \text{ and } \frac{4}{9} \times 9 = 4 \text{ Feet, the Solidity.}$$

EXAM

EXAMPLE II.

WHAT is the solid Content of a Tree whose Length is 31 Feet, and the Girt 16 Inches ?

16 Inches = 1,3 Feet. $1,3 \times 1,3 \times 31 = 54,07$ Feet, the Solidity.

EXAMPLE III.

SUPPOSE a Piece of Timber $9\frac{1}{2}$ Feet long, and 39 Inches in the Girt ; what is the Solidity ?

39 Inches = 3,25 Feet. Then $3,25 \times 3,25 \times 9,5 = 102,984375$, or 103 Feet, nearly.

EXAMPLE IV.

WHAT is the Solidity of a Tree whose Girt is 21 Inches, and the Length $40\frac{1}{2}$ Feet ?

21 Inches = ,916 Feet. Then $,916 \times ,916 \times 40,5 = 34,04475$ Feet, or 34 Feet, nearly.

EXAMPLE V.

A TREE whose Girt is 31 Inches, and the Length 24 Feet ; what is the Solidity ?

31 Inches = 2,583 Feet. Then $2,583 \times 2,583 \times 24 = 160,16$ Feet, the Solidity.

THOUGH this Method for it's Ease is commonly made Use of by Artificers, yet it is absolutely false ; for the fourth Part of the Circumference of a Circle cannot be equal to the Side of a Square whose Area shall be equal to the Area

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Area of that Circle: For if the Circumference of a Circle be 1, the Side of a Square of equal Area to that Circle will be ,2821 ; whereas by the false Method of the Girt it is but ,25 ; therefore,

MULTIPLY the Compass of the Tree by ,2821, and it reduces it to the Side of a Square of equal Area with a Circle whose Circumference is equal to the Tree's Compass, which Side square, and multiply the Square by the Length, and it gives the true Content, or Solidity.

TAKE the second of the foregoing Examples, in order to compare the Difference of the Methods, viz.

THE Length 31 Feet, and the Girt 16 Inches ; what is the Solidity ?

16 \times 4 = 64 Inches, which is 5,3 Feet, the Compass. Then 5,3 \times ,2821 = 1,50453 Feet, the Side of a square Equal. And 1,50453 \times 1,50453 \times 31 = 70,172, &c. the true Solidity. But by the common Way the Solidity is 55 Feet. Therefore 70,172 — 55 = 15,172 Feet, &c. a Difference too considerable to be neglected.

THE Solidity would be nearly the same, by saying as 11 : is to 14 :: so is the Solidity found by the common Way : to the true Solidity.

OR the Solidity may be thus found.

MULTIPLY the Square of the Compass by ,07958, and this Product by the Length, will give the Solidity required ; for ,07958 is the Area of a Circle whose Circumference is 1.

TAKE Example the fifth, viz. A Tree 24 Feet long, and Girt 31 Inches ; what is the Solidity ?

By the common Method the Solidity was found to be 160,16 Feet ; then,

First,

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First, To find the true Solidity from the erroneous one.

11 : 14 :: 160,16 : 203,84, &c. The Solidity.

Secondly, By the Square Equal to the Tree's Compass.

$31 \times 4 = 124$ Inches, and 124 Inches = $10,3$ Feet, the Tree's Compass.

$10,3 \times 2821 = 2,91503$, the Side of the Square Equal.

$2,91503 \times 2,91503 \times 24 = 203,938064026$, the Solidity.

Thirdly, By the Area of a Circle whose Circumference is equal to the Tree's Compass.

$10,3 \times 10,3 \times ,07958 \times 24 = 203,937013$, the Solidity.

HERE the Answer turns out almost 204 Feet, by these three different Methods ; but I advise that either of the two latter Methods be used.

E X A M P L E VI.

SUPPOSE the Length of a Tree to be 36,9 Feet, and the Compass 87 Inches ; what is the Solidity ?

First, By the Square Equal.

87 Inches = 7,25 Feet.

THEN, $7,25 \times ,2821 = 2,045225$, the Side of the Square.

AND $2,045225 \times 2,045225 = 4,1829453$, &c. the Square Equal.

Secondly, $7,25 \times 7,25 \times ,07958 = 4,18292375$, the Circle Equal.

Now either of these multiply'd by the Length, gives the Solidity.

THUS

of MENSURATION. 143

THUS the Circle equal, viz. 4,18292375
 $\times 36,75 = 153,7224478125$ Feet, the Solidity
 sought.

I THINK the Circle Equal in most Operations
 is found with less Trouble.

WHEN rough Timber (that is, Timber with
 the Bark on) is to be measur'd for Sale; there is
 generally some Allowance made for the Bark or
 Rind, which is one tenth Part of the Circum-
 ference; that is, subtract .1 (one tenth) of the
 Tree's Compass, from the Compass, and it re-
 duces it to such a Compass, as the Artificers (among
 whom this Method is used) suppose hath no Bark.
 But this Rule is only used in the Allowance for
 Oak-Bark: for the Allowance for Elm, Beach,
 Ash, &c. must be smaller.

IF a Tree have any Branches that measure two
 Feet in Compass, such Branches are called Timber
 and their Solidity is to be computed, and added to
 that of the Tree.

EXAMPLE VII.

THERE is a Tree, whose Length is 18 Feet,
 Compass $3\frac{1}{2}$ Feet, and hath two Boughs, the Length
 of one 10 Feet, and Compass $2\frac{1}{2}$ Feet, and the
 Length of the other is 12 Feet, and Compass 2 Feet,
 and being Oak, there is an Allowance to be made
 for Bark; what is the Solidity of the whole?

$$\frac{3,5}{10} = ,35$$

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$$\frac{3,5}{10} = ,35, \text{ and } 3,5 - ,35 = 3,15.$$

$$\frac{2,5}{10} = ,25, \text{ and } 2,5 - ,25 = 2,25.$$

$$\frac{2,0}{10} = ,2, \text{ and } ,2 - ,2 = 1,8.$$

So 3,15, 2,25, and 1,8 are the respective Compasses without Bark.

THEN, $3,15 \times 3,15 \times ,07958 \times 18 = 14,211$, &c. the Tree's Solidity.

AND, $2,25 \times 2,25 \times ,07958 \times 10 = 4,027$, &c. the first Bough's Solidity.

AND, $1,8 \times 1,8 \times ,07958 \times 12 = 3,094$, &c. the second Bough's Solidity.

THEN, $14,211 + 4,027 + 3,094 = 21,332$ Feet, the whole Solidity.

THE foregoing Method of girding the Tree at an equal Distance from each End, is only to be used when the Tree is nearly of the same Thickness in every Part; but if it be very irregular, the Method then, is to gird it in several Places, (as many as is thought necessary) and the several Compasses together, and their Sum divide by the Number of Girts (which is thought by some) will give the mean Circumference.

Note, THE Buyer hath Liberty to gird the Tree any where between the Middle and the Ground-End if he chuses it.

EXAMPLE

EXAMPLE.

SUPPOSE a Tree to be so irregular, that it is thought necessary to gird it in four several Places ; the first Compass is 5,5 Feet ; the second is 4,7 Feet ; the third is 3,6 Feet, and the Fourth is 4,2 Feet ; the Length is 15 Feet ; what is the Solidity thereof ?

$5,5 + 4,7 + 3,6 + 4,2 = 18$, and $\frac{18}{4} = 4,5$ Feet, the mean Compass.

THEN, $4,5 \times 4,5 \times ,07958 \times 15 = 24,173925$, the Solidity.

IF at any Time you would know how many Loads there are in any Quantity of Timber ; find the solid Feet as before directed. Then if the Timber is hewn, divide the Feet by 50, gives the Loads ; but if the Timber is unhewn, divide the Feet by 40, gives the Loads.

FOR the Measurers say that the Allowances given to 40 Feet of rough Timber, makes it equal to 50 Feet of hewn Timber, which is accounted to weigh a Ton or 20 hundred Weight, and so much is reckon'd a Cart Load.

WHAT has been already done, only concerns Timber, whose Ends are equal, or nearly so ; but in the Books of Mensuration there are other Sorts of figur'd Timber considered ; such as unequal round Timber ; squar'd, or unequal square Timber the common Methods of calculating the Solidity of such Timber is as follows :

First, OF squared Timber ; that is, such Timber that when the Sides are hewn away, they are parallel, and the Areas of the Ends are equal ; all such Timber is measured thus :

O

FIND

FIND the Area of one End, which Area multiply by the Length of the Timber, and it gives the Solidity.

Secondly. OF unequal squared, and unequal round Timber; that is, such Timber, that when hew'd into Form, have the Areas of their Ends unequal, that is, when the Dimensions of the Figure at one End are different from those of the other End.

THE common Methods of measuring such Timber is; by finding a mean Girt as before directed.

BUT if such Pieces of Timber are the Frustums of Pyramids or Cones, they had much better be measured as such, by the Rule hereafter given; and therefore I think it needless to give any Examples, or say any thing more of them in this Place; for the Directions and Examples already given, take in all the Cases that can possibly occur in the four Varieties, *viz.* Equal and Unequal, Squared, or Round; and otherwise than which, Measurers in their calculating the Solidity of Timber very seldom consider.

S E C T

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Dep
5
and

SECTION III.

Of divers RIGHT-LINED *and* CIRCULAR
FIGURES.

PROPOSITION II.

To find the Solidity of any Prism.

RULE.

FIND the Area of the Base (or one End) which
Area multiply by the Length, and it gives the
Solidity.

EXAMPLE I.

WHAT is the Solidity of a Cube whose Side is
12 Inches?

$12 \times 12 = 144$, the Area of the End, and 144
 $\times 12 = 1728$, the Inches in a solid Foot.

EXAMPLE II.

WHAT is the Solidity of a Block of Marble,
whose Length is 10 Feet, Breadth $5\frac{3}{4}$ Feet, and
Depth $3\frac{1}{2}$ Feet?

$5.75 \times 3.5 = 20.125$, the Area of the End,
and $20.125 \times 10 = 201.25$ Feet, the Solidity.

EXAMPLE III.

WHAT is the Solidity of a Triangular Prism, whose Length is 18 Feet, and one Side of the equilateral Base is $1\frac{1}{2}$ Feet?

IN the Table of PROPOS. VII. of *Superf. Meas.* the Area of an equilateral Triangle, whose Side is 1, is ,433014; therefore, $1,5 \times 1,5 \times ,433014 = ,97427925$, the Area of the End.

THEN, $,97427925 \times 18 = 17,5370265$, the Solidity.

EXAMPLE IV.

WHAT is the Solidity of a Cylinder, whose Length is 5 Feet, and the Diameter of the Base is 2 Feet? *Fig. 3.*

$2 \times 2 \times ,7854 = 3,1416$, the Area and $3,1416 \times 5 = 15,708$ Feet, the Solidity.

IT is customary in the Books of Mensuration, to give a Rule separate, for the working of each of the foregoing Examples; but I do not see the Necessity of it, as they all fall under one Definition, and may be wrought by one single Rule, viz. the foregoing one.

PROPOSITION III.

To find the Solidity of any Pyramid or Cone.
(Fig. 4, 5.)

R U L E.

FIND the Area of the Base, which Area multiply by one third Part of the Altitude, and it gives the Solidity; because every Pyramid or Cone is one third of a Prism of the same Base and Altitude.

EXAMPLE I.

WHAT is the Solidity of a Pyramid, whose Height is 24 Feet, and the Side of it's square Base is 3 Feet?

$3 \times 3 = 9$, the Area of the Base.

AND, $\frac{24}{3} = 8 = \frac{1}{3}$ of the Height.

THEN, $9 \times 8 = 72$, the Solidity sought.

EXAMPLE II.

WHAT is the Solidity of a Pyramid whose Height is 15 Feet, and one Side of it's Hexagonal Base is 18 Inches?

IN the Table to PROP. VII. of *Superf. Meas.* the Area of a Hexagon, whose Side is 1, is found to be 2,598076.

THEN, $1,5 \times 1,5 \times 2,598076 = 5,845671$, the Area of the Base.

THEN, $5,845671 \times \frac{15}{3} = 29,228355$, the Solidity.

EXAMPLE III.

WHAT is the Solidity of a Cone, the Diameter of whose Base is 18 Inches, and the Altitude 15 Feet?

18 Inches = 1,5, and $1,5 \times 1,5 \times ,7854 = 1,76715$, the Area of the Base.

AND, $\frac{15}{3} = 5$, the Height.

THEN, $1,76715 \times 5 = 8,83575$ Feet, the Solidity.

PROPOSITION IV.

To find the Superficies of any right Cylinder.

RULE.

MULTIPLY the Circumference of the Base by the Length, and the Product is the Curve Superficies, to which add the Area of each End, and the Sum is the whole Superficies. For the Curve Superficies of every right Cylinder is equal to the Area of a Parallelogram, whose Length is equal to the Circumference, and the Breadth equal to the Height or Length of the Cylinder; for the Length multiply'd by the Breadth, gives the Area of the Parallelogram; and if this Parallelogram be circularly bent, till the two bounding End-Lines meet, it will form the Curve of the Cylinder.

EXAMPLE

EXAMPLE.

WHAT is the Superficies of a Cylinder whose Circumference is 100 Inches, and the Length 14 Feet?

100 Inches = 8,3 Feet, and $8,3 \times 14 = 116,6$ Feet, the Curve Superficies.

$8,3 \times 8,3 \times ,07958 \times 2 = 11,0537$, the Area's of the Ends.

THEN, $116,6 + 11,0537 = 127,7204$, the whole Superficies.

PROPOSITION V.

To find the Superficies of any right Cone.

RULE.

MULTILLY the Length of the slant Side by half the Circumference of the Base, and it gives the Curve Superficies; to which add the Area of the Base, and it gives the whole Superficies.

FOR the Curve Superficies of every right Cone is equal to the Area of a Sector of a Circle, whose Radius is equal to the slant Side of the Cone, and the Sector Arch, equal to the Circumference of the Cone's Base. For the Sector's Radius multiply'd by half it's Arch, will give it's Area; and if the Sector be circularly bent, till the Radii coincide, it will form the Curve Superficies of the Cone.

EXAMPLE

EXAMPLE

WHAT is the Superficies of a right Cone whose Circumference at the Base is 24 Feet, and the Length of the slant Side 32 Feet?

$$32 \times \frac{24}{2} = 384, \text{ the Curve Superficies.}$$

AND, $24 \times 24 \times ,07958 = 45,93808$, the Area of the Base.

THEN, $384 + 45,93808 = 429,93808$, the whole Superficies.

PROPOSITION VI.

To find the Solidity of the Frustrum of a Pyramid.
(Fig. 7.)

RULE.

MULTIPLY the Sides of the greater and lesser Ends together, and to the Product add one third of the Square of the Difference of those Sides, which reduces the Ends to the Square of a mean Side; which Square multiply'd by it's proper Multiplier, taken out of the Table of Polygons Areas, (*Page* 106) will give the mean Area, which multiply by the Frustrum's Length gives the Solidity.

EXAMPLE

EXAMPLE I.

WHAT is the Solidity of the Frustum of a square Pyramid, one Side of the greater End being 18 Inches, that of the lesser End 15 Inches, and the Height 5 Feet?

18 Inches = 1,5 Feet, and 15 In. = 1,25 Feet.

THEN, $1,5 - 1,25 = ,25$, and $,25 \times ,25 = ,0625$, and $\frac{,0625}{3} = ,0208\bar{3} = \frac{1}{50}$ of the Square of the Side's Difference.

AND, $1,5 \times 1,25 = 1,875$, and $1,875 + ,0208\bar{3} = 1,8958\bar{3}$, and $1,8958\bar{3} \times 5 = 9,4791\bar{6}$ Feet, the Solidity sought.

EXAMPLE II.

WHAT is the Solidity of a Hexagonal Pyramid's Frustum, the Side of whose square End is 3 Feet, that of the lesser End 2 Feet, and the Length 12 Feet?

$3 - 2 = 1$, and $1 \times 1 = 1$, and $\frac{1}{3} = ,3 = \frac{1}{3}$ of the Square of the Side's Difference.

AND $3 \times 2 = 6$, and $6 + ,3 = 6,3$, and $6,3 \times 2,598076$ (the Area of a Hexagon, whose Side is 1, Page 106) the Product is 16,4544813, the mean Area.

AND $16,4544813 \times 12$ (the Length) gives 197,453776, the Solidity required.

IF a Pyramid's Frustum's Ends be of any other Figure than that of a regular Polygon. First find the Side of a Square, whose Area shall be equal to the Area of the greater End, and the same for the lesser End; and then work as the foregoing Rule directs.

P R O-

PROPOSITION VII.

To find the Solidity of the Frustum of a Cone.

RULE.

To the Sum of the Squares of the Diameter of the End, add the Product of those Diameters; multiply this Sum by $\frac{1}{3}$ of the Height, and this Product by ,7854, and it gives the Solidity. *Fig. 8.*

Note. THIS Rule will serve for the Solidity of the Frustum of a Pyramid, omitting the Multiplication by ,7854.

AND the Rule given for a Pyramid will serve for a Cone, by multiplying the Result given by that Rule by ,7854.

EXAMPLE.

WHAT is the Solidity of the Frustum of a Cone, the Diameter of the greater End being 4 Feet, that of the lesser End 2 Feet, and the Altitude 9 Feet?

$4 \times 4 = 16$, and $2 \times 2 = 4$, and $16 + 4 = 20$, the Sum of the Squares of the Diameters.

$4 \times 2 = 8$, the Rectangle of the Diameters.

AND, $\frac{9}{3} = 3 = \frac{1}{3}$ of the Height.

$20 + 8 = 28$, and $28 \times 3 \times ,7854 = 65,9736$ Feet the Solidity.

By these two last Propositions should all unequal'd squared or round Timber be measured; for most
Trees

Trees being bigger at the Ground-End than at the other, may be considered (when first cut down, and the Branches lopt off) as the Frustrums of Cones; and if the Sides are cut square (or squarish) they may then be considered as the Frustrums of Pyramids, and consequently in either Case, they should be measured according to the Figure they represent, supposing them to be regular; but if the Difference of the Ends be small, there is no need of having Recourse to any other Directions than those given in the first Proposition; but as all Pyramids and Cones are considered as having their Sides perfectly strait, most Trees will differ from them, by Reason of the Inequality of their Sides or Girths.

PROPOSITION VIII.

To find the Altitude of any Pyramid or Cone, having the Dimensions of any Frustrum of that Pyramid or Cone given.

R U L E.

SAY, as the Difference between the two Diameters given (if a Cone) or between one Side of the greater End, and one Side of the lesser End, (if a Pyramid) is to the Height of the Frustrum, so is the greater Diameter (if a Cone) or one Side of the greater End (if a Pyramid) to the whole Height.

EXAM-

E X A M P L E.

THERE is a Frustum of a Cone, the Diameter of the greater End is 83 Inches, and that of the lesser End is 54 Inches, and the Altitude is 12 Feet; what was the Altitude of the whole Cone?

$83 - 54 = 29$ Inches, which is 2,416 Feet, the Difference of the Diameters, and 83 Inches $= 6,916$ Feet.

THEN, $2,416 : 12 :: 6,916 : 34,3448$, &c. the Height of the whole Cone as required.

PROPOSITION IX.

To find what Length from the Vertex of a Pyramid or Cone will answer to any given Solidity

R U L E.

SAY as the Solidity of the whole Pyramid or Cone, is to the Cube of it's Altitude; so is any given Part of the Solidity, to the Cube of it's Altitude reckon'd from the Vertex downwards, whose Cube-Root is the Length.

EXAM-

EXAMPLE.

I HAVE a conical Piece of Timber, the Diameter of the Base is 18 Inches, and the Length 12 Feet; what Distance from the Vertex must I apply the Saw, to cut it into two Pieces of equal Solidity?

18 Inches = 1,5 Feet. Then $1,5 \times 1,5 \times \frac{12}{3} \times 7854 = 7,0686$ Feet, the whole Solidity (per PROP. III.) half of which is 3,5343, and $12 \times 12 \times 12 = 1728$, the Cube of the Altitude.

THEN, $7,0686 : 1728 :: 3,5343 : 864$, the Cube of the Length of half the Solidity measured from the Vertex, whose Cube-Root is 9,5, &c. and so far from the Vertex must the Saw be apply'd to cut it into two equal Pieces.

PROPOSITION X.

To find the Solidity of the Hoofs or Ungula of the Frustum of a Pyramid. [Fig. 7.]

IF it is of any other Form than a square Pyramid, it must first be reduced to such, by finding the Side of a Square of equal Area, Then

R U L E.

To the Square of the Side of one End add one half, the Rectangle of the Sides of the two Ends, and this Sum multiply by one third of the Height gives the Solidity.

P

Note.

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Note. You will have the Solidity of the greater or lesser Ungulæ, according to the End of the Fruustum you use.

EXAMPLE.

THERE is the Fruustum of a square Pyramid, one Side of the greater End is 1,5 Feet, the Side of the lesser End is 1,25 Feet; and the Height is 5 Feet; what is the Solidity of each Ungulæ?

$1,5 \times 1,5 = 2,25$, the Square of the greater Side.

AND, $1,5 \times 1,25 = 1,875$, and $\frac{1,875}{2} = ,9375$,

and $2,25 + ,9375 = 3,1875$, and $\frac{5}{3} = 1,6$, then

$3,1875 \times 1,6 = 5,125$, the Solidity of the greater Hoof.

AGAIN, $1,25 \times 1,25 = 1,5625$, and $1,5625 + ,9375 = 2,5$, and $2,5 \times 1,6 = 4,16$, the Solidity of the lesser Ungulæ.

AND, $5,125 + 4,16 = 9,47916$, the Solidity of the Fruustum given.

IF this Example and the Result be compared with the Example and the Result in PROP. V. you will see how the Result of the Work agrees; for there the same Example is given to find the Solidity at once, which turns out the same as this does where it is found at two Operations.

PROPOSITION XI.

To find the Solidity of the Hoofs of a Cone's Frustum.

First, FOR the greater Hoof. Fig. 8.

R U L E.

To the Square of the greater Diameter, add half the Product of the two Diameters, and to this Sum add the Difference of the Diameters; multiply this Sum by the Height, and this Product by ,2618, gives the Solidity.

Secondly, FOR the lesser Hoof.

R U L E.

To the square of the lesser Diameter, add half the Product of the two Diameters; from this Sum subtract the Difference of the Diameters; multiply this Difference by the Height, and this Product by ,2618, gives the Solidity.

EXAMPLE.

THERE is a Cone's Frustum, the Diameter of the greater End is 4 Feet, that of the lesser End 2 Feet, and the Height 9 Feet; what is the Solidity of each Hoof?

$4 \times 4 = 16$, and $4 \times 2 = 8$, and $\frac{8}{2} = 4$, and $4 - 2 = 2$, then $16 + 4 + 2 = 22$, and $22 \times 9 \times ,2618$ ($\frac{1}{2}$ of 3,1416) = 51,8364, the Solidity of the greater Hoof.

AGAIN, $2 \times 2 = 4$, and $4 \times 2 = 8$, and $\frac{8}{2} = 4$, and $4 - 2 = 2$, then $4 + 4 + 2 = 10$, and $10 \times 9 \times ,2618 = 23,562$, the Solidity of the lesser Hoof.

AND, $51,8364 + 23,562 = 75,3984$, the Solidity of the whole Frustum, which is exactly the same with the Solidity found by the Rule for finding the Solidity of a Frustum, which may be seen by comparing this with the Example to Proposition VI.

Note. There is a Mistake in the Rule given in WARD'S *Young Mathematician's Guide*; for the Multiplication should be by the whole Height, and not by one third of the Height as there given.

SECT X

SECTION IV.

Of a SPHERE and it's Parts.

PROPOSITION XII.

To find the Superficies of a Sphere or Globe.
[Fig. 6.]

RULE.

MULTIPLY the Area of it's Circumscribing Circle by 4, and the Product is the Superficies.

EXAMPLE.

WHAT is the Superficies of a Bomb-Shell, whose Diameter is 16 Inches?

16 Inches = 1,3 Feet, then $1,3 \times 1,3 \times ,7854 = 1,3962\phi$ Feet, the Area of the circumscribing Circle.

AND, $1,3962\phi \times 4 = 5,5850\phi$, the Superficies.

OR thus: Multiply the Sphere's Axis into it's Circumference, gives the Superficies.

FOR, $1 : 3,1416 :: 1,3 : 4,1888$, and $4,1888 \times 1,3 = 5,5850\phi$, as before.

PROPOSITION XIII.*To find the Solidity of a Sphere.***RULE.**

FIND the Solidity of a Cylinder the Diameter of whose Base, as also it's Height are each equal to the Sphere's Axis, which Solidity multiply by 2, and the Product divide by 3, gives the Solidity of the Sphere; for every Sphere is equal to two thirds of it's circumscribing Cylinder.

OR thus: If you suppose the Sphere to be made up of an innumerable Number of Cones, whose Vertices all meet at the Center, and the Areas of whose Bases form the Superficies of the Sphere now as the Solidity of any one Cone is equal to the Area of it's Base, multiply'd by one third of it's Height, therefore the Superficies of all the Cone Bases, multiply'd by the one third of the Height of one, gives the Solidity of all the Cones. But one third of the Height of one Cone is one sixth of the Sphere's Axis, and the Superficies of the Bases of all the Cones is equal to the Superficies of the Sphere, and the Solidity of all the Cones equal to the Solidity of the Sphere; therefore,

RULE.

MULTIPLY the Superficies of the Sphere by one sixth of the Axis, and the Product is the Solidity.

EXAMPLE

EXAMPLE.

WHAT is the Solidity of a Sphere, whose Diameter is 30 Inches?

30 Inches = 2,5 Feet, the Axis.

First. By the first Rule. $2,5 \times 2,5 \times ,7854 = 4,90875$, the Area of the Cylinder's Base.

AND, $4,90875 \times 2,5 = 12,271875$, the Cylinder's Solidity.

THEN, $12,271875 \times 2 = 24,54375$, and $\frac{24,54375}{3} = 8,18125$ Feet, the Sphere's Solidity.

Secondly. By the second Rule. $2,5 \times 2,5 \times ,7854 = 4,90875$, the Area of the circumscribing Circle.

AND, $4,90875 \times 4 = 19,635$, the Sphere's Superficies.

AND, $\frac{2,5}{6} = ,41\bar{6} = \frac{1}{2}$ of the Axis.

THEN, $19,635 \times ,41\bar{6} = 8,18125$, the Sphere's Solidity as before.

BUT, the Solidity of a Sphere may be more easily found, by finding the Solidity of a Sphere whose Diameter is 1, thus:

$1 \times 1 \times ,7854 = ,7854$, the Cylinder's Base.

AND, $,7854 \times 1$ (the Cylinder's Height) = $,7854$, the Cylinder's Solidity.

THEN, $,7854 \times 2 = 1,5708$, and $\frac{1,5708}{3} = ,5236$, the Sphere's Solidity.

THEN Spheres being in Proportion to each other as are the Cubes of their Diameters.

THEREFORE, As the Cube of 1 : is to ,5236, it's Solidity, so is the Cube of any other Sphere's Diameter to it's Solidity; that is,

RULE

R U L E.

MULTIPLY the Cube of the Axis by ,5236, and the Product is the Solidity.

As in the foregoing Example where the Diameter is 30 Inches or 2,5 Feet. $2,5 \times 2,5 \times 2,5 \times ,5236 = 8,18125$ Feet, the Solidity, the same with the Solidity found by the two foregoing Rules.

If by having the Circumference of a Sphere, you would find the Solidity; the best Way would be to find the Solidity of a Sphere, whose Circumference is 1; thus,

$3,1416 : 1 :: 1 : ,318309$, the Diameter.

AND, $\frac{,318309}{2} \times \frac{1}{2} = ,07958$, the Base's Area.

AND, $,318309 \times ,07958 = ,025331$, &c. the Solidity of the Cylinder.

AND, $\frac{,025331}{3} = ,0084436$, which \times by 2 = ,016887, the Sphere's Solidity, whose Circumference is 1; then,

R U L E.

MULTIPLY the Cube of the Circumference by ,016887, gives the Solidity.

PROPOSITION XIV.

To find the Superficies of any Segment of a Sphere.

First, THE Axis and versed Sine (or Height of the Segment) being given,

R U L E.

SAY, as the Length of the Sphere's Axis is to the Sphere's Superficies, so is the versed Sine to the Curve Superficies of the Segment, to which add the Area of it's Base, and it gives the Superficies of the whole Segment.

Secondly, BY knowing the Diameter of the Segment's Base, and the versed Sine.

R U L E.

TO the Area of the Segment's Base, add the Area of a Circle whose Radius is equal to the versed Sine, and the Sum is the Curve Superficies, to which add the Area of the Base, gives the whole Superficies.

EXAMPLE

EXAMPLE.

THERE is a Sphere whose Axis is 21 Inches ; I want to know the Superficies of a Segment of this Sphere, cut off at 6 Inches from the Center ?

OPERATION *by the first Rule.*

$$\frac{21}{2} = 10,5, \text{ and } 10,5 - 6 = 4,5, \text{ the versed Sine.}$$

$21 \times 21 \times ,7854 \times 4 = 1385,4456$, the Superficies of the whole Sphere, *per* PROPOS. IX.

As $21 : 1385,4456 :: 4,5 : 296,8812$, the Curve Superficies of the Segment.

OPERATION *by the second Rule.*

By the 14th Prop. of *Sup. Meas.* the Diameter of the Segment's Base will be 17,23368 ; then $17,23368 \times 17,23368 = 296,999$ &c. or 297 and $297 \times ,7854 = 233,2638$ the Area of the Segment's Base.

AND $4,5 \times 2 = 9$; then $9 \times 9 \times ,7854 = 63,6174$ the Area of a Circle whose Radius is the versed Sign.

THEN $233,2638 + 63,6174 = 296,8812$ the Curve Superficies of the Segment, the same as by the first Rule.

THEN $296,8812 + 233,2638 = 530,145$, the Superficies of the whole Segment, as was required.

PROPOSITION XV.

To find the Solidity of any Segment of a Sphere.

[Fig. 9.]

First, By having the versed Sign, and the Diameter of the Segment given,

R U L E.

To three times the Square of half the Segment's Base add the Square of the versed Sign ; multiply this Sum by the versed Sign, and this Product by ,5236, and the Product is the Solidity.

Secondly, By having the Axis, and versed Sine given.

R U L E.

From three times the Axis subtract twice the versed Sine ; multiply the Remainder by the Square of the versed Sine, and this Product multiply by ,5236, and the Product is the Solidity.

E X A M P L E I.

WHAT is the Solidity of the Segment of a Sphere, the Diameter of the Segment's Base being 17,23368 inches, and it's Height 4,5 ?

$$\frac{17,23368^2}{2} = 8,61684, \text{ and } 8,61684 \times 8,61684$$

74,2499, or $74,25$, and $74,25 \times 3 = 222,75$ 3 times the Square of half the Segment's Base.

AND, $4,5 \times 4,5 = 20,25 =$ Square of the versed Sine.

THEN, $222,75 + 20,25 = 243$, and $243 \times 4,5 \times ,5236 = 572,5566$, the Segment's Solidity.

EXAMPLE

EXAMPLE II.

THE Axis 21, what is the Solidity of the Segment of that Sphere, whose Height is 4,5 ?

$21 \times 3 = 63$, and $4,5 \times 2 = 9$, and $63 - 9 = 54$, the Difference between three times the Axis, and twice the versed Sine.

THEN, $4,5 \times 4,5 = 20,25$, and $20,25 \times 54 \times ,5326 = 872,5566$, the Segment's Solidity equal to that found by the former Rule.

THUS you may observe from the Examples to the four last Propositions, that the Result turns out the same (in those Examples where the same thing is sought after) although operated by different Rules.

PROPOSITION XVI.

To find the Solidity of the middle Frustum of a Sphere, otherwise called a Zone; that is, a Sphere lessened by two Segments cut parallel to each other. [Fig. 10.]

First. SUPPOSE the Zone cut through the Center parallel to the Ends, and call each Piece a Segment of the Zone; then will each Segment have two Ends, the Diameter of the greater being equal to the Sphere's Axis.

RULE.

To twice the Area of the greater End, add the Area of the lesser End; and multiply the Sum by one third of the Distance of the Ends, gives the Solidity of that Segment of the Zone; or thus,

RULE

R U L E.

FIND the Solidity of two Cylinders, the Height of each equal to the Height of the Segment, and the Diameter of ones Base equal to the greater End's Diameter of the Zone's Segment, and the Diameter of the others Base equal to the Diameter of the lesser End of the Zone's Segment, subtract one third of the Difference of the Solidity of these Cylinders, from the Solidity of the greater Cylinder, and the remainder is the Solidity of that Segment of the Zone.

E X A M P L E.

SUPPOSE the Axis of a Sphere is 20 Inches ; what is the Solidity of a Zone, the Section being made 6 Inches from the Center, and the Diameter at the Section being 16 Inches?

By the first Rule.

20 Inches = 1,6 Feet, and 16 Inches = 1,3 Feet, and 6 Inches = ,5 Feet. $1,6 \times 1,6 \times ,7854 \times 2 = 4,348$ = twice the Area of the greater End. $1,3 \times 1,3 \times ,7854 = 1,39626$ = lesser End's Area.

AND, $\frac{,5}{3} = ,16 = \frac{1}{6}$ of the Height.

THEN, $4,348 + 1,39626 = 5,7396$, and $5,7396 \times ,16 = ,9566$, the Solidity.

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By the second Rule.

$1,6 \times 1,6 \times ,7854 \times ,5 = 1,08587$, the greater Cylinder's Solidity.

AND $1,3 \times 1,3 \times ,7854 \times ,5 = ,69813 =$ lesser Cylinder's Solidity.

AND $1,08587 - ,69813 = ,3877$, and $\frac{3877}{3} = ,12927 = \frac{1}{3}$ the Difference of the Cylinder's Solidity.

THEN, $1,08587 - ,12927 = ,9566$, the Solidity as before.

SECTION V.

PRACTICAL QUESTIONS.

HERE follows a Collection of Questions contrived to exercise most of the foregoing Propositions, which (and many others of a like Nature) may be easily solved, if what has been delivered in the foregoing Pages be well understood.

QUESTION I.

How many 3 Inch Cubes connected together, will make a 12 Inch Cube? *Fig. 11.*

$\frac{12}{3} = 4 =$ Number of 3 Inch Cubes to make one Side of a 12 Inch Cube.

THEN, $4 \times 4 \times 4 = 64$, the Number of 3 Inch Cubes requisite to make a 12 Inch Cube.

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QUESTION II.

A FARMER borrow'd of a Neighbour of his a Piece of a Haycock, which measured 6 Feet every way (that is a Cube whose Side was 6 Feet) and the borrowing Farmer paid back two equal cubical Pieces, each of whose Side were three Feet ; Query whether the lending Farmer was fully paid ?

$6 \times 6 \times 6 = 216$, the Solidity of the Piece borrowed.

$3 \times 3 \times 3 = 27$, and $27 \times 2 = 54$, the Solidity of the Pieces paid. Then,

$\frac{216}{54} = 4$, therefore the lending Farmer was paid but a fourth.

QUESTION III.

I WANT an Iron Roller for a Garden, whose outside Diameter shall be 20 Inches, Length of the Roller 50 Inches, and Thickness of the Metal $1\frac{1}{2}$ Inch ; now supposing every Cube Inch weigh $4\frac{1}{2}$ Ounces, what will the whole come to at $3\frac{1}{2}$ per lb ?

$1\frac{1}{2} \times 2 = 3$, the double Thickness, and $20 - 3 = 17$, the Inner Diameter.

$20 \times 20 \times 17,7854 = 314,16$, and $17 \times 17 \times 17,7854 = 226,9806$, and $314,16 - 226,9806 = 87,1794$, and $87,1794 \times 50 = 4358,97$, the Solidity of the Roller.

$4\frac{1}{2} \text{ oz.} = 265625 \text{ lb.}$ and $1 \text{ lb.} : 265625 \text{ lb.} :: 4358,97 \text{ lb.} : 1157,8514 \text{ lb.}$ the Weight
 $3\frac{1}{2} \text{ d.} = 10135416 \text{ l.}$ and $1 \text{ lb.} : 10135416 \text{ l.} :: 1157,8514 : 15,679 \text{ l.}$ &c. $= 15 \text{ l. } 13 \text{ s. } 7 \text{ d.}$
 the whole Value of the Roller.

QUESTION IV.

I WANT two Pyramidal Lamp-Posts of Stone ; I would have the Bases square, and each set on a Pedestal of 3 Feet in Height ; the Height of the Pyramid must be 7 Feet, and the Side of it's Base 16 Inches, which is 2 Inches less than the Side of the Pedestal it stands on ; what will they both come to at 2 s. 6 d. per Foot ? *Fig. 12.*

16 Inches + 2 = 18 Inches = 1,5 Feet, one Side of the Base of the Pedestal.

AND, 16 Inches = 1,3 Feet, $1,5 \times 1,5 \times 3 = 6,75$ Feet, the Solidity of one Pedestal.

AND, $1,3 \times 1,3 \times \frac{7}{3} = 4,0\frac{1}{3}$, the Solidity of one Pyramid.

THEN $6,75 + 4,0\frac{1}{3} = 10,77\frac{1}{3}$, the Solidity of one Post.

AND, $10,77\frac{1}{3} \times 2 = 21,54$, the Solidity of both.

THEN, 2 s. 6 d. = ,125 l. and 1 F. : ,125 l. : : 21,54 F. : 2,698 l. = 2 l. 13 s. 10 $\frac{1}{2}$ d. the whole Expence.

QUESTION V.

I HAVE given Orders for a Stone Cone to be made, to put in the midst of a Fountain in my Garden ; the Height was to be 24 Feet, and the Diameter of the Base 12 Feet, and I have agreed with the Workman for 3 s. per Foot solid ; but my Mind altering, I intend to have an Octagonal Pyramid of the same Height, and one Side of the Base to be 5 Feet ; what will be the Difference of the Costs, supposing the Price to be the same in both ?

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$12 \times 12 \times ,7854 = 113,0976 =$ Area of the Base.

AND, $113,0976 \times \frac{24}{3} = 904,7808$ Feet, the Cone's Solidity.

THEN, $1 F. : 15 l. :: 904,7808 : 135,71712 l.$ the Cone's Expence.

$5 \times 5 \times 4,828427 = 120,780675 =$ Area of the Base.

AND, $120,71675 \times \frac{24}{3} = 965,6854$, the Pyramids Solidity.

THEN, $1 F. : 15 l. :: 965,6854 F. : 144,85281 l.$ the Pyramid's Expence.

THEN, $144,85281 l. - 135,71712 l. = 9,13569 l. = 9 l. 2 s. 8 \frac{1}{2} d.$ the Difference of the Costs.

QUESTION VI.

WHAT will the Painting a conical Church Spire come to at $8 d. \text{ per Yard}$, supposing the Circumference of the Cone's Base to be 64 Feet and the Altitude 118 Feet?

$1 : ,318309 :: 64 : 20,371776$, the Diameter of the Base.

$\frac{20,371776}{2} = 10,185888$, the Radius.

AGAIN, $118 \times 118 = 13924$, and $10,185888 \times 10,185888 = 103,7523$, &c. and $13924 + 103,7523 = 14027,7523$, whose Square Root is 118,43 Feet, the slant Side.

AND $118,43 \times \frac{64}{2} = 3789,76$ Feet, and $\frac{3789,76}{9} = 421,084$ Yards.

AND $1 Yd. : ,03 l. :: 421,084 Yds. : 14,03614 l. = 14 l. 0 s. 8 \frac{1}{2} d.$ and so much will be the whole Expence.

QUESTION VII.

I HAVE in my Garden an Octagonal Pyramid, with a Cube fixed to it's Vertex ; one Side of the Cube is equal to one Side of the Pyramid's Base, which is 9 Inches, and the Pyramid's Height is 7 Feet: I desire to know what the Guilding this Pyramid and Cube will come to, at 2 *d. per Inch* ?

$9 \times 9 \times 6 = 486$, the Superficies of the Cube, and 7 Feet = 84 Inches.

IN order to find the Superficies of the Pyramid, the Length from the Vertex to the middle of the Sides must be found, which will be the Altitude of one Triangle ; but before the Length can be found, the Perpendicular Height of one of the Triangles composing the Base must be found. Thus,

$9 \times 9 \times 4,828427 = 391,102587$, the Area of the Base.

AND, $\frac{391,102587}{8} = 48,887823 =$ the Area of one of the Triangles composing the Base ; the Base of which Triangle is the Side of the Polygon ; therefore, $\frac{48,887823}{4,5} = 10,86396$, the Altitude of one

of the Triangles composing the Base. Then the Height $84 \times 84 = 7056$, and $10,86396 \times 10,86396 = 117,025626$, &c. and $7056 + 117,025626 = 7173,025626$, &c. whose square Root is 84,69, &c. the Height of one of the Triangles composing the Pyramid. Then $84,69 \times \frac{9 \times 8}{2} = 3048,84 =$ the Pyramid's Superficies. And $3048,84 + 486 = 3516,84$,

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= 3516,84, the whole Superficies of the Pyramid and Cube. Then,

1 In. : ,0083 l. : : 35,1684 In. : 29,307 l. = 29 l. 6 s. 1½ d. the whole Expence.

QUESTION VIII.

I HAVE made a Marble Frustum of a Cone, for which I am to have 12 s. per Foot solid ; the Diameter of the greater End is 4 Feet, that of the lesser End 1,5 Feet, and the Length of the slant Side 8 Feet ; what is the Frustum worth ?

$\frac{4}{2} - \frac{1,5}{2} = 1,25$, and $8 \times 8 = 64$, and $1,25 \times 1,25 = 1,5625$, and $64 - 1,5625 = 62,4375$, whose square Root is 7,9 Feet the Altitude of the Frustum.

THEN, $4 \times 4 = 16$, and $1,5 \times 1,5 = 2,25$; and $4 \times 1,5 = 6$, also, $\frac{7,9}{3} = 2,63 = \frac{2}{3}$ the Altitude.

AND, $16 + 2,25 + 6 = 24,25$, and $24,25 \times 2,63 \times ,7854 = 50,154335$ Feet, the Solidity of the Frustum.

AND, 1 F. : ,6 l. : : 50,154335 F. : 30,092601 l. = 30 l. 1 s. 10½ d. the Expence of the whole.

QUEST.

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QUESTION IX.

SUPPOSE a Church-Spire was to be built of an octagonal Form, one Side of the greater End to be 24 Feet, one Side of the lesser End to be 12,5 Feet, and the Height 80 Feet; but the Inside of the Spire is to be run up in a conical Form, the Diameter of the Base is to be 56 Feet, and the Diameter at Top to be 28 Feet; I desire to know the Expence at 4 s. 6 d. per Foot Solid?

$12,5 \times 24 = 300$, the Rectangle of the Sides of the Ends.

$24 - 12,5 = 11,5$ and $11,5 \times 11,5 = 132,25$,

and $\frac{132,25}{3} = 44,08\bar{3} =$ to $\frac{1}{3}$ of the Square of the Difference of the two Sides.

AND, $44,08\bar{3} \times 4,828427 = 1661,38125691\bar{6}$, the mean Area.

AND, $1661,38125691\bar{6} \times 80 = 132910,500553$, or 132910,5 Feet, Pyramid's Frustum's Solidity.

$56 \times 28 = 1568$, and $56 - 28 = 28$, and $28 \times 28 = 784$, and $\frac{784}{3} = 261,3$, and $1568 + 261,3 = 1829,3$, and $1829,3 \times 80 \times ,7854 = 114930,672$, the Solidity of the contain'd Cone.

THEN, $132910,5 - 114930,672 = 17979,828$ Feet, the Solidity of the Stone Work.

THEN, 1 F. : ,225 l. :: 17979,828 F. : 4045,4613 l. = 4045 l. 9 s. 2 $\frac{1}{2}$ d. the Cost.

QUEST.

QUESTION X.

A MASON is employ'd to compleat a Cone, whose Vertex by Accident is struck off; he having made the upper Part level, the Dimensions of the Frustrum are as follows: Length of the slant Side 12 Feet, Diameter of the upper End 6 Feet, and the Circumference of the Base 38 Feet; what will it cost at 3 s. *per Foot*, to put a Piece on, equal to that taken off?

As 1 : ,318309 :: 38 : 12,095742, or 12,1, the Diameter of the Base. $12,1 - 6 = 6,1$, and $\frac{6,1}{2} = 3,05$, the half Difference of the Diameters.

Then $12 \times 12 = 144$, and $3,05 \times 3,05 = 9,3025$, and $144 - 9,3025 = 134,6975$, whose square Root is 11,6, &c. the Frustrum's Height. Then $6,1 : 11,6 :: 6 : 11,4$, &c. the Height of the Piece

wanting; and $6 \times 6 \times ,7854 \times \frac{11,4}{3} = 107,44272$, &c. Feet, the Solidity of the Piece wanting. Then $1 F. : ,15 l. :: 107,44272, F. &c. : 16,1164$, = 16 l. 2 s. 4 d. the whole Expence to compleat the Cone.

QUESTION XI.

THREE Men bought a tapering Piece of Timber, which was the Frustrum of a square Pyramid; one Side of the greater End was 3 Feet, and one Side of the

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the lesser End 1 Foot, and the Length 18 Feet ; and they paying equally, are to have equal Shares ; what is the Length of each Man's Piece ? *Fig. 13.*

First, $3 \times 1 = 3$, and $3 - 1 = 2$, and $2 \times 2 = 4$, and $\frac{4}{3} = 1,3$, and $3 + 1,3 = 4,3$, and $4,3 \times 18 = 78$, the Solidity of the Frustum, and $\frac{78}{3} = 26$, the Solidity of each Man's Share. As $2 : 18 :: 1 : 9$, the Height of a Piece wanting ; and $1 \times 1 \times \frac{9}{3} = 3$, the Solidity of the Piece wanting to compleat the Pyramid ; then $26 + 3 = 29 =$ one Man's Solidity added to the Top, and $26 \times 2 = 52$, and $52 + 3 = 55 =$ two Mens Solidity added to the Top, and $78 + 3 = 81$, the Solidity of the whole Pyramid, and $18 + 9 = 27 =$ to it's Height ; and $27 \times 27 \times 27 = 19683$, the Cube of the Pyramid's Altitude. Then $81 : 19683 :: 29 : 6923,543$, &c. whose Cube Root is 19,0 ; and $81 : 19683 :: 55 : 13365$, whose Cube Root is 23,7. Then $27 - 23,7 = 3,3$ *F.* the first Man's Length, and $23,7 - 19 = 4,7$ *F.* = second Man's Length, and $19 - 9 = 10$ *F.* = third Man's Length, accounting from the greater End for the first Man's, &c.

QUESTION XII.

THE Dimensions of a Cone's Frustum are as follows: Length of the slant Side 40 Inches, Diameter of the greater Base 50 Inches, and the Diameter of the lesser Base 20 Inches ; if this Frustum was to be cut into two equal Pieces parallel to the Base ; what would the gilding each Piece come to at 2*d.* per square Inch ?

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50 — 20 = 30, and $\frac{30}{2} = 15$, and $40 \times 40 = 1600$, and $15 \times 15 = 225$, and $1600 - 225 = 1375$, whose square Root is 37,0, the Height of the Frustrum. Then as $(30 - 20) 30 : 37 :: 20 : 24,6$, the Height of the Piece wanting, and $37 + 24,6 = 61,6$, the whole Cone's Height. And

$50 \times 50 \times \frac{61,6}{3} \times ,7854 = 40360,83$, the whole

Cone's Solidity. And $20 \times 20 \times \frac{24,6}{3} \times ,7854 = 2583,093$, the Solidity of the Piece wanting.

And $40360,83 - 2583,093 = 37777,74$, the Solidity of the given Frustrum. And $\frac{37777,74}{2}$

$= 18888,84$, and $18888,84 + 2583,093 = 21471,963$, the Solidity of half the Frustrum added to the Solidity of the Piece wanting to make the Cone. And $61,6 \times 61,6 \times 61,6 = 234504,68$, the Cube of the whole Cone's Altitude. And

$40360,83 : 234504,68 :: 21471,963 : 124756,454$, &c. whose Cube Root is 49,9, &c. or 50 Inches, and $50 - 24,6 = 25,3$, the Length of half the Frustrum's Solidity measur'd from the small End.

AGAIN, as the whole Cone's Height 61,6 : it's Diameter at the Base 50 :: the upper Frustrum and Piece wanting, their Heights : to Diameter of the Frustrum's Section.

As $37 : 40 :: 24,6 : 26,6$, slant Side of the Piece wanting; and $40 + 26,6 = 66,6$, the whole Cone's slant Side; and $50 : 66,6 :: 40,5 : 54$, the slant Side of the upper Frustrum and Piece wanting. The Curve Superficies of the first Cone is 5236, and the Area of it's Base 1963,5; that of the second Cone 3435,3396, and it's Base's Area 1288,24235; that

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that of the third Cone 837,76, it's Base's Area 314,16. Then $5236 - 2435,3396 = 1800,6604$, and $1800,6604 + 1963,5 + 1288,24235 = 5052,40275$, the whole Superficies of the lower Piece.

AND, $34353396 - 837,76 = 2597,5796$, and $2597,5796 + 1288,24235 + 314,16 = 4199,98195$, the whole Superficies of the upper Piece of the given Frustum. And $5052,40275 + 4199,98195 = 9252,3847$. Then $1 \text{ In.} : ,0083 \text{ l.} :: 9252,3847 : 77,1032$, &c. $= 77 \text{ l. } 2 \text{ s. } 0 \frac{1}{4} \text{ d.}$ the whole Expence.

QUESTION XIII.

WHAT is the Weight of a Cannon-Ball of 7 Inches in Diameter supposing one of 4 Inches weighs 9 lb?

$4 \times 4 \times 4 = 64$, and $7 \times 7 \times 7 = 343$, and $64 : 9 \text{ lb} :: 343 : 48,2 \text{ lb}$, the Ball's Weight: For it will always be as the Cube of 4 is to it's Weight 9 lb, so is the Cube of any other Ball's Diameter to it's Weight.

QUESTION XIV.

WHAT is the Diameter of a Ball that weighs 72 lb?

As $9 \text{ lb} : 64 :: 72 \text{ lb} : 512$, whose Cube Root is 8, the required Diameter: For it will always be, as $9 : 64$, so is any Weight given to the Cube of the Diameter, whose Root will be the Diameter.

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QUESTION XV.

SUPPOSE the Globe or Ball on the Top of St Paul's Church to be 6 *F.* in Diameter; what did the Gilding thereof come at $3\frac{1}{2}$ *d.* per Inch square.

$1 : 3,1416 :: 72 \text{ In.} : 226,1952 \text{ In.}$ the Circumference, and $226,1952 \times 72 = 16286,0544$, the Superficies. And $3\frac{1}{2} = ,014583$. Then, $1 \text{ In.} : ,014583 \text{ l.} :: 226,1952 : 237,504 \text{ l.}$ &c. $= 237 \text{ l. } 10 \text{ s. } 1 \text{ d.}$ the Expence.

QUESTION XVI.

WHAT is the Weight of a Bomb-Shell, that is 16 Inches Diameter without the Metal, which suppose is 3 Inches, admitting every Cubic Inch weighs $4\frac{1}{2}$ Ounces?

$16 \times 16 \times 16 \times ,5236 = 2144,6656$, and $16 - 3 \times 2 = 10$, and $10 \times 10 \times 10 \times ,5236 = 523,6$, and $2144,6656 - 523,6 = 1621,0656$, and as $1 \text{ C. I.} : ,28125 \text{ lb} :: 1621,0656 : 465,9247 \text{ lb}$ the Weight.

QUESTION XVII.

I HAVE a Cylindric Vessel the Depth is 28 Inches, and the Diameter of the Base is 46 Inches; but I want another that shall hold just as much again, whose Depth shall be 3 Feet; what must be the Diameter?

$46 \times 46 \times ,7854 \times 28 = 46533,3792$, the Contents of the given Vessel. And $46533,3792 \times 2 = 93066,7584$, the Contents of the required Vessel. And $\frac{93066,7584}{36 \text{ (3 F.)}} = 2585,1877$, the Area of the required Vessel's Base.

$1 : 1,2732 :: 2585,1877 : 3292,460725$, whose square Root is 57,3797, the Diameter of the Vessel's Base, as required.

R

QUESTION

QUESTION XVIII.

I HAVE a Grainery 47 Feet 8 Inches long, 18 Feet 5 Inches broad, and 9 Feet 7 Inches high; but I want another that will hold four times as much, and have the Dimensions in the same Proportion to each other as the old one hath; what will be the Dimensions of the new one?

47 Feet 8 Inches = 47,8, and 18 Feet 5 Inches = 18,418, and 9 Feet 7 Inches = 9,588. Then $18,418 \times 9,588 \times 47,8 = 8412,835648x$, the Solidity of the old Grainery. And $8412,835648x \times 4 = 33651,342592x$, the Solidity of the new one. And $47,8 \times 47,8 \times 47,8 = 108303,98$, the Cube of the Length of the old one.

$8412,835648x : 108303,98 :: 33651,342592x : 433215866, &c.$ whose Cube Root is 75,6, &c. the Length of the new one.

$47,8 : 75,6 :: 18,418 : 29,2, &c.$ the Breadth of the new one.

$47,8 : 75,6 :: 9,588 : 15,2$ nearly, the Depth or Height of the new one.

QUESTION XIX.

I HAVE a Vault to be dug; whose Length must be 20 Feet, Breadth 12 Feet, and Depth 7 Feet; and I am to pay the Workman 4 s. 6 d. per Floor for Work and Carriage; what will the Digging cost me?

$12 \times 7 \times 20 = 1680$ Feet, the Contents, and $\frac{1680}{324} = 5,188$ Floors. Then 1 Fl. : ,225 l. :: 5,188 Fl. : 1,166625 l. = 1 l. 3 s. 4 d.

Note, 18 Feet square, and 1 Foot deep, or 324 Feet solid, is called a Floor.

QUESTION

QUESTION XX.

I REMEMBER once to have met with a Cone's Frustum, whose Solidity was 65,9736 Feet, and the Height was 9 Feet ; the Diameter was in the Proportion of 2 to 1 ; what were those Diameters ?

$\frac{65,9736}{9} = 7,3304$, the mean Area. As $1 : 1,2732 :: 7,3304 : 9,33306528$, the Square of the Diameter of the mean Area. And $2 \times 1 = 2$, and $2 - 1 = 1$, and $1 \times 1 = 1$, and $\frac{1}{3} = ,3$, and $2,3 = a$ mean Square between the Squares of the given proportional Numbers 2 and 1. Then,

As $2,3 : 9,33306528 :: 4(2 \times 2) : 16$, whose square Root is 4, the greater Diameter ; and $2 : 1 :: 4 : 2$, the lesser Diameter.

BUT all Questions of this Nature may be expeditiously solved by the following Rule. But first,

OF those two Numbers given in the Proportion as the Diameter sought, call the greater the *Antecedent*, and the lesser the *Consequent*. Then,

R U L E.

MULTIPLY the given Solidity by the Square of the Antecedent, for a Dividend : To the Rectangle of the Antecedent and Consequent add the Square of each, and this Sum multiply by the Height,

R 2

and

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and this Product by ,2618 ($\frac{1}{3}$ of ,7854) for a Divisor ; then Division being made, the Quotient, will be the Square of the greater Diameter, whose Root will give the Diameter. Take the foregoing Example.

$2 \times 2 = 4$, and $65,9736 \times 4 = 263,8944$, the Dividend.

$2 \times 2 = 4$, and $1 \times 1 = 1$, and $2 \times 1 = 2$, and $4 + 1 + 2 = 7$, and $7 \times 9 \times ,2618 = 16,4934$, the Divisor. Then, $\frac{263,8944}{16,4934} = 16$, whose square Root is 4, the greater Diameter, and $2 : 1 :: 4 : 2$, the lesser Diameter.

QUESTION XXI.

SUPPOSE two Porters having a Quart of strong Beer between them, agree to drink it off at two Pulls, that is, a Draught to each ; now the first having given it the black Eye as they call it, that is, drank till the Surface of the Liquor touch'd the opposite Edge of the Bottom, he gave the remaining Part of it to the other ; what was the Difference of their Shares ? Supposing the Quart Pot was the Frustum of a Cone ; the Depth being 5,7 Inches, the Diameter at Top 3,7 Inches, and the Solidity 70,5 solid Inches ?

FIRST, the Diameter of the Bottom must be found, and then the Difference of the Solidities of the two Hoofs.

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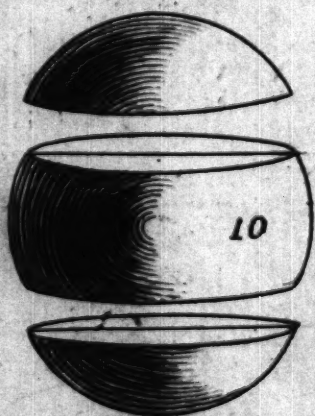
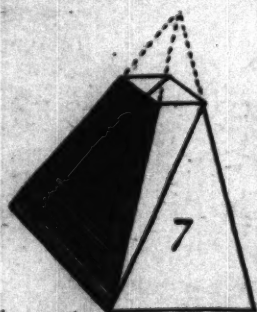
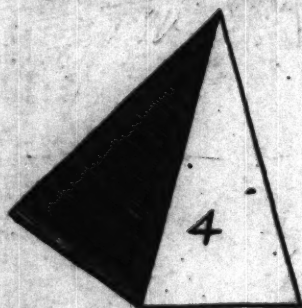
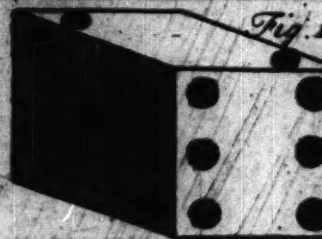
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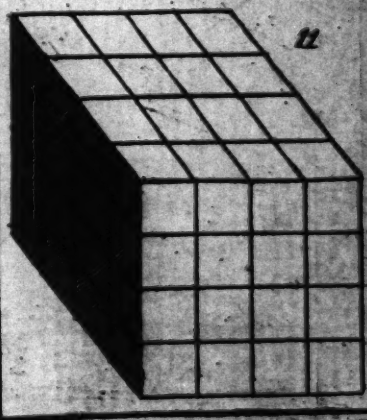
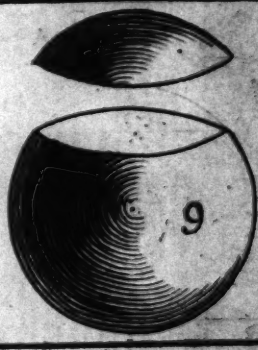
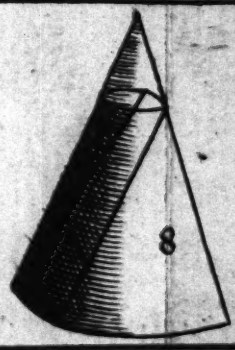
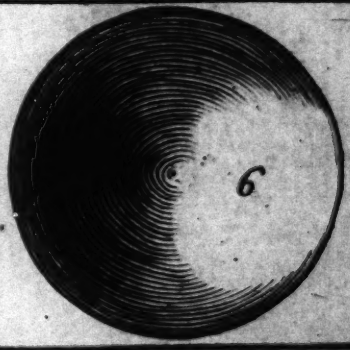
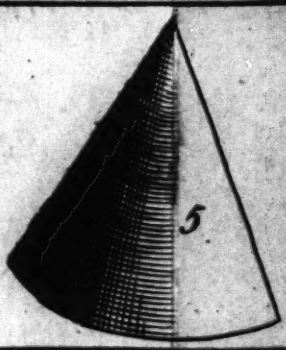
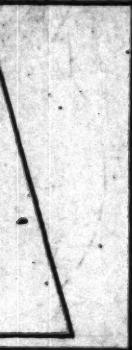
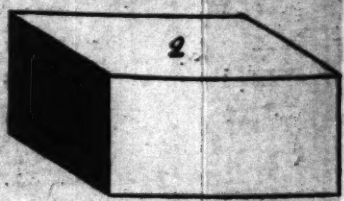
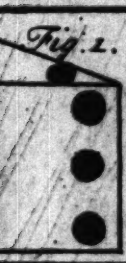
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Plate. 11.







of MENSURATION 185

HAVING the Solidity, Length, and one of the Diameters of any Cone's Frustum given, the other Diameter may be found by the following.

R U L E.]

MULTIPLY the Length by ,2618, and divide the Solidity by this Product: From this Quotient subtract $\frac{1}{4}$ of the Square of the given Diameter; and extract the Square Root out of the Remainder, from which Root if half the given Diameter be subtracted, the Remainder will be the Diameter sought.

,2618 \times 5,7 = 1,49226, and $\frac{70,5}{1,49226}$
 = 47,243, &c. and $3,7 \times 3,7 \times 3 = 41,07$,
 and $\frac{41,07}{4} = 10,2675 = \frac{1}{4}$ of the Square of
 the given Diameter.

AND $47,243 - 10,2675 = 36,9755$, whose Square Root is 6,080, &c.

AND, $\frac{3,7}{2} = 1,85$. Then $6,08 - 1,85$
 = 4,23, the Diameter of the Bottom of the Pot.

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AGAIN, For the lesser Hoof, or the Quantity the first Man drank.

$$3,7 \times 3,7 = 13,69, \text{ and } 4,23 \times 3,7 = 15,651, \\ \text{and } \frac{15,651}{2} = 7,8255. \text{ And } 13,69 + 7,8255$$

$$= 21,5155. \text{ And } 4,23 - 3,7 = ,53, \text{ and } \\ 21,5155 - ,53 = 20,9855. \text{ And } 20,9855 \\ \times 5,7 \times ,2618 = 31,2158, \text{ \&r. Inches, what} \\ \text{the first Man drank.}$$

$$\text{AND } 70,5 - 31,2158 = 39,2842 \text{ Inches,} \\ \text{what the last Man drank.}$$

$$\text{THEN } 39,2842 - 31,2158 = 8,0684 \text{ cubic} \\ \text{Inches, the Difference of their Shares.}$$

End of the Second Part.

APPEN-



APPENDIX.

HERE I shall shew the Measurement of some Curves, called *the Conic Sections*, and also of the Solids, generated by their Rotation on their Axes.

SECTION I.

DEFINITIONS.

I. IF a Cone be cut into two Pieces, in such a Manner, that the cutting Instrument (which is supposed flat) do always keep parallel to the Base; it is plain that the Surface of each Piece is a Circle.

II. BUT

II. BUT if the cutting Instrument passes thro' the Cone, so as always to keep parallel to the Side opposite that Side on which the Instrument entered; the Surface of each Piece at this Section is called a *Parabola*.

III. IF the Cone be so cut, that the Instrument passes evenly through any where between the Sections of the Circle and Parabola, the Surface of each Piece at this Section is called an *Ellipsis*.

IV. IF the Cone be so cut, that the Instrument passes evenly any where through between the Section of the Parabola, and that Side of the Cone on which the Instrument entered, the Surface of each Piece at this Section is called a *Hyperbola*.

THUS, suppose ABC (*Fig. 1. Plate 3.*) a Cone, and D the Point on which the Instrument is to enter.

First. THE Section DE is a *Circle*.

Secondly. THE Section DF is an *Ellipsis*.

Thirdly. THE Section DG is a *Parabola*.

Fourthly. THE Section DH is a *Hyperbola*.

THE Forms of the Curves of the *Ellipsis*, *Parabola*, and *Hyperbola*, are represented in the Figures 2, 3, and 4.

V. THAT Point of the Curve in any of the conic Sections, where the Curve is most acute, [or sharpest] is called the *Vertex*.

VI. A LINE drawn through the Vertex, so as to divide the Area of the Section into two equal Parts, is called the *Axis of the Curve*.

Note. IN the *Ellipsis* this Line is called the *Transverse Diameter*, or *Axis*.

VII. A LINE drawn at right Angles to the Axis, and terminated at each End by the Curve is called an *Ordinate*.

Note. THAT an Ordinate drawn through the middle of the Transverse Axis in the Ellipsis, is called the *Conjugate Axis*, or *Diameter*.

VIII. IF an Ellipsis be moved round on either of it's Diameters [supposing it fixed] till the Motion end where it began; a Solid that would fill the Space thus generated, is called a *Spheroid*.

Note. IF the Ellipsis be turned on the Transverse Axis, the Solid generated is called a *Prolate Spheroid*, and is nearly of the Figure of an Egg.

BUT if it be turned on the Conjugate Axis, the Solid generated is called an *Oblate Spheroid*, and is nearly of the Figure of a Bowling-Green Bowl.

IX. SUPPOSE the Axis of a Parabola or an Hyperbola be fixed, and then the Curve be turned round it's Axis, till the Motion end where it began; a Solid that would fill the Space thus generated is called a *Conoid*.

IN the Parabola it is called a *Parabolic Conoid*, or *Paraboloid*.

IN the Hyperbola it is called a *Hyperbolic Conoid*, or *Hyperboloid*.

X. IF a Parabola be turned round on an Ordinate [fixed] till the Motion end where it began; a Solid that would fill the Space thus generated, is called a *Parabolic Spindle*.

SECTION II.

Of the ELLIPSIS and it's Parts.

PROPOSITION I.

Having the Transverse and Conjugate Diameter of an Ellipsis given, to find the Area.

R U L E.

MULTIPLY the Rectangle contained by the Transverse and Conjugate Diameters by ,7854, and the Product is the Area.

FOR the Area of every Ellipsis is a mean Proportional between the Area of two Circles, one whose Diameter is the Transverse Axis, and the other the Conjugate Axis. *Fig. 4.*

EXAMPLE I.

WHAT is the Area of an Ellipsis, whose Transverse Axis is 24, and the Conjugate 18 ?

$24 \times 18 \times ,7854 = 339,2928$, the Area sought.

HENCE

HENCE it is easy to conceive that if the Area of any Ellipsis be divided by ,7854 and that Quotient by one of the Diameters, the Result will be the other Diameter.

THE Segment of any Ellipsis is obtain'd thus.
Fig. 5.

IF the Ordinate cutting off the Segment be parallel to the Transverse Axis, describe a Circle on the Conjugate Axis, and then it will be :

As the Area of this Circle, is to the Area of the Ellipsis, so is the Area of any Segment in this Circle, to the Area of it's corresponding elliptical Segment.

EXAMPLE.

THERE is an Ellipsis whose Transverse Diameter is 12 Inches, and the Conjugate 8 Inches, I desire to know the Area of a Segment of this Ellipsis cut 2 Inches distant from the Center, and parallel to the Transverse Axis:

Now it is evident, if a Circle be described on the Conjugate Axis, the Semi-chord in the Circle will be less than the Semi-ordinate in the Ellipsis; in order to find the Area of the Segment of an Ellipsis, there must be known the Ellipsis's Area, the Circle's Area, and the Area of the Circle's Segment corresponding to that of the Ellipsis.

$$12 \times 8$$

$12 \times 8 \times 7854 = 75,3984$, the Area of the Ellipsis.

$8 \times 8 \times 7854 = 50,2656$, the Area of a Circle described on the conjugate Axis.

$\frac{8}{2} = 4$, the Radius. And $4 - 2 = 2$, the versed Sine. And $4 + 2 = 6$.

THEN $6 \times 2 = 12$, whose Square Root is 3,46, &c. the half Chord, *per* PROPOSIT. XIV. PART. I.

$4 \times 8 \times 2 = 64$, and $14 \times 4 \times 4 = 224$, and $224 - 64 = 160$, and $2 \times 2 \times 6 = 24$, and $160 - 24 = 136$, and $4 \times 9 = 36$, and $2 \times 6 = 12$, and $36 + 12 = 48$, and $\frac{136}{48} = 2,87$.

THEN $2,87 \times 3,46 = 9,803$, the Area of the Segment of the Circle, *per* PROPOSIT. XVII. PART. I.

$50,2656 : 75,3984 :: 9,803 : 14,7$, the Area of the Elliptical Segment.

AFTER the same Manner would the Area be found, was the Ordinate to be given parallel to the Conjugate; only in this Case you must work with the Circle described on the Transverse Axis. *Fig. 6.*

PRO-

PROPOSITION III.

To find the Solidity of a Spheroid.

(FIG. 7.)

R U L E.

FIND the Solidity of a Cylinder, whose Axis is equal to that of the Spheroid, and the Diameter of whose Base is equal to the greatest Ordinate in the Spheroid, and $\frac{2}{3}$ of that Cylinder's Solidity, is the Spheroid's Solidity.

OR thus, Multiply the Square of the greatest Ordinate by the Axis; and this Product by ,5236 gives the Spheroid's Solidity.

E X A M P L E.

WHAT is the Solidity of a Spheroid, whose Axis is 24 Inches, and the greatest Ordinate 18 Inches?

$18 \times 18 \times ,7854 \times 24 = 6107,2704$, the Solidity of the Cylinder.

AND, $6107,2704 \times 2 = 12214,5408$, and $12214,5408 \times \frac{2}{3} = 4071,5136$, the Spheroid's Solidity.

OR thus, $18 \times 18 \times 24 \times ,5236 = 4071,5136$, the Solidity as before.

S B E C A U S E

BECAUSE a Spheroid is $\frac{2}{3}$ of the circumscribing Cylinder, and a Cone is $\frac{1}{3}$ of a Cylinder of the same Base and Altitude, it follows that a Spheroid is double to a Cone, whose Axis is equal to the Spheroid's Axis, and the Diameter of whose Base is equal to the Spheroid's greatest Ordinate.

PROPOSITION III.

To find the Solidity of any Frustum of a Spheroid. [Fig. 8.]

R U L E.

SAY as the Solidity of the inscribed or circumscribing Sphere : is to the Solidity of the Spheroid :: so is any Segment of a Sphere : to the corresponding Part of the Spheroid.

THIS Rule will either find the Solidity of the middle Frustum or Zone, or the Solidity of a Frustum, cut from one End of one Axis, and parallel to the other Axis.

EXAMPLE I.

SUPPOSE a Spheroid whose transverse Axis was 20 Inches, and the Conjugate 16 Inches,

was

was to have a Slice taken off, parallel to the Conjugate, at 6 Inches distant from the Center; what is the Solidity of that Piece?

$16 \times 16 \times 20 \times ,5236 = 2680,832$, the Solidity of the Spheroid.

$20 \times 20 \times 20 \times ,5236 = 4188,8$, the Solidity of the Sphere.

$\frac{20}{2} = 10$, and $10 - 6 = 4$, the versed Sine.

AND $20 \times 3 = 60$, and $4 \times 2 = 8$, and $60 - 8 = 52$, and $4 \times 4 = 16$.

THEN $52 \times 16 \times ,5236 = 435,6352$, the Solidity of a Segment of the circumscribing Sphere, corresponding to the Solidity of the required Spheroid's Segment.

THEN, $4188,8 : 2680,832 :: 435,6352 : 278,80$, &c, the Spheroid's Segment.

BUT if the Solidity of the Zone was required, besides the foregoing Rule there is another much more expeditious as follows. *Fig. 9.*

R U L E.

To twice the Square of the conjugate Diameter, add the Square of the Diameter of the Zone's End; this Sum multiply by the Distance between the Conjugate and End, and this last Product by $\frac{,5236}{2}$ (,2618) and the Product is the Solidity of that Part of the Zone.

If the Diameters of the Zone's two Ends are equal, double this last Product, gives the Solidity of the whole Zone. But if they are unequal, proceed as before, with the Zone's other End, and the two Solidities added, give the whole Solidity.

THIS Rule is of Use in finding the Solidity or Contents of a Cask, supposed to be the middle Frustum of a Spheroid. *Fig. 9.*

EXAMPLE II.

SUPPOSE a Cask (the middle Frustum of a Spheroid) the Diameter at the Bung (or Conjugate) is 16, the Diameter of either of the Heads (or Ends) is 12,8, and the Length is 12; what is the Content in solid Measure?

THIS Example which is the middle Frustum of the Spheroid in the foregoing Example, (whose Transverse is 20) I shall operate by both the Rules.

First, By the first Rule.

$20 \times 20 \times 20 \times ,5236 = 4188,8$, the Sphere's Solidity.

$16 \times 16 \times 20 \times ,5236 = 2680,832$, the Solidity of the Spheroid, as before.

$20 \times 20 \times ,7854 \times 2 = 628,32$, and $16 \times 16 \times ,7854 = 201,0624$, and $628,32 + 201,0624 = 829,3824$, and $829,3824 \times \frac{12}{3} = 3317,5296$,

the Solidity of the circumscribing Sphere's Zone, per THEOR. XIII. PART II.

THEN $4188,8$ Spher. S. : $2680,832$ Spheroid's S. : : $3317,5296$ Spher. Z. S. : $2123,218944$, the Solidity of the Cask, or Spheroid's Zone.

Secondly,

Secondly, By the second Rule.

$16 \times 16 \times 2 = 512$, the double Square of the Conjugate, or Bung Diameter.

$12,8 \times 12,8 = 163,84$, and $512 + 163,84 = 675,84$, and $675,84 \times \frac{12}{2} = 4055,04$, and $4055,04 \times ,2618 \times 2 = 2123,218944$, the Cask's Solidity as before.

EXAMPLE III.

WHAT is the Solidity of a Cask, taken as the middle Zone of a Spheroid; supposing the Bung Diameter 3 Feet, the Head 2 Feet, and the Length 5 Feet?

$3 \times 3 \times 2 = 18$, and $2 \times 2 = 4$, and $18 + 4 = 22$, and $22 \times 5 \times ,2618 = 28,798$ Feet, the Solidity sought after.

Note, THE foregoing Rule is calculated for finding the Solidity of but one Side of the Zone, supposing it cut through at the Conjugate; but if the two Ends are equal, instead of multiplying by the Distance between the Conjugate and End, multiply by the Zone's whole Length, &c. (i. e. Breadth) and the Product will be the whole Solidity.

SECTION III.

Of a PARABOLA and it's Parts.

PROPOSITION IV.

To find the Area of a Parabola, by having the Ordinate and Axis given. [Fig. 10]

RULE.

MULTIPLY the Ordinate by the Axis, and this Product by 2, and the last Product divide by 3, and the Quote is the Area.

FOR every Parabola is $\frac{2}{3}$ of it's circumscribing Parallelogram.

EXAMPLE.

What is the Area of a Parabola, whose Axis is 12, and the Ordinate 16?

$16 \times 12 \times 2 = 384$, and $\frac{384}{3} = 128$, the Area sought after.

PRO-

PROPOSITION V.

To find the Solidity of a Parabolic Conoid, by having the Diameter of the Base and the Altitude given. [Fig. 11]

R U L E.

MULTIPLY the Area of the Base by half the Height, and the Product is the Solidity; for every parabolic Conoid is half of it's circumscribing Cylinder.

E X A M P L E.

WHAT is the Solidity of a Paraboloid, the Diameter of whose Base is 20, and the Height 12?

$20 \times 20 \times ,7854 \times \frac{12}{2} = 1884,76$, the Solidity sought after.

PROPOSITION VI.

To find the Solidity of the lower Frustum of a Paraboloid. [Fig. 12]

R U L E.

MULTIPLY the Sum of the Squares of the Diameters of the two Ends, by ,3927 ($= \frac{1}{8}$ of 3,1416) and multiply the Product by the Length, gives the Solidity.

EXAM-

EXAMPLE.

A FRUSTUM of a Paraboloid has one Diameter at the End 40, and the other End Diameter is 24, and the Length between the Diameters, or Height, is 12; what is the Solidity?

$40 \times 40 = 1600$, and $24 \times 24 = 576$, and $1600 + 576 = 2176$, and $2176 \times .3927 \times 12 = 10254.1824$, the Solidity sought after.

PROPOSITION VII.

To find the Solidity of a Parabolic Spindle, by having the Length and the Diameter of the greatest circumscribing Circle given.
[Fig. 13]

RULE.

MULTIPLY the Square of the Diameter by
 $.418 (i. e. \frac{2 \times 3,1416}{15})$ and this Product multiply by the Length, gives the Solidity.

EXAMPLE.

EXAMPLE.

THERE is a parabolic Spindle, whose Length is 9 Feet, and the Diameter of the greatest Circle is 40 Inches ; what is the Solidity ?

40 In. = 3,3 F. Then $3,3 \times 3,3 \times ,418 \times 9 = 418,7$ Feet, the Solidity.

Note, EVERY Parabolic Spindle is $\frac{1}{15}$ of it's circumscribing Cylinder.

PROPOSITION VIII.

To find the Solidity of the middle Frustum, or Zone of a Parabolic Spindle. [Fig. 14]

RULE.

To twice the Square of the greater Diameter add the Square of the lesser Diameter, and this Sum multiply by ,7854 ; from this Product subtract $\frac{2}{3}$ of the Area of a Circle whose Diameter is equal to the Difference between the greater and lesser End's Diameters ; and this Remainder multiply by $\frac{1}{3}$ of the Length gives the Solidity.

THIS

THIS Proposition is of Use in finding the Contents of such Casks whose Figure is that of the middle Zone of a Parabolic Spindle.

EXAMPLE.

THERE is a Cask whose Figure is the middle Zone of a parabolic Spindle ; the Bung Diameter 44 Inches, the Head Diameter 28 Inches, and the Length 5 Feet ; what is the Solidity or Contents thereof ?

44 In. = 3,6 F. and 28 In. = 2,3 F. and $3,6 - 2,3 = 1,3$ F. the Difference between the Diameters.

THEN $3,6 \times 3,6 = 13,4$, and $13,4 \times 2 = 26,8 =$ twice the Square of the Bung Diameter.

$2,3 \times 2,3 = 5,4$ the Square of the Head Diameter.

AND $26,8 + 5,4 = 32,2$, and $32,2 \times ,7854 = 25,3946$, and $1,3 \times 1,3 \times ,7854 = 1,3962,6$ and $\frac{1,39626 \times 2}{5} = ,558506$, and $25,3946$

$- ,558506 = 24,86093$, and $24,86093 \times \frac{5}{3} (1,6) = 41,41015$ Feet, the Solidity.

SECT.

SECTION IV.

Of the HYPERBOLA.

THERE may several Hyperbolical Curves be drawn through the same Vertex, and all pass through the Extremities of the same Ordinate, kept always at the same Distance from the Vertex.

HENCE it will be very easy to conceive, that no settled Rules can be given for finding the Area of an Hyperbola [like those for the Ellipsis and Parabola] without taking into Consideration some other Lines beside the Axis and Ordinate, which I shall omit, this Treatise not being intended to instruct the Reader in the Properties of the Conic Sections.

BUT here note, that a certain ingenious Author hath asserted, that every Appolonian (or Conical) Hyperbola is greater than half, and less than three fourths of it's circumscribing Parallelogram: If so, it will be accurate enough to take all Hyperbola's at a mean; thus, say every Hyperbola is $\frac{3}{4}$ of it's circumscribing Parallelogram; therefore multiply the Product of the Axis and Ordinate by 5, and this Product divide by 8; will give the Area nearly.

EXAM-

EXAMPLE.

WHAT is the Area of a Hyperbola, whose Axis is 28, and the Length of the Ordinate drawn that Distance from the Vertex is 44?

$44 \times 28 \times 5 = 6175$, then $\frac{6175}{8} = 771,875$, the Area.

As there can be no Rules given (whose Praxis is easy) for finding the Area of an Hyperbola, so there can be no easy Rules given for finding the Solidities of the Solids, that may be generated by the Rotation of the Hyperbola round it's Axis or Ordinates.

SECTION V.

Of REGULAR SOLIDS.

THOSE Solids whose Sides being all equal, and the several Superficies bounding the Solid all alike and equal, and which would be so contain'd within a Sphere, that it's Angles would all touch the Superficies of the Sphere, are called *Regular Solids*.

THERE are five regular Solids.

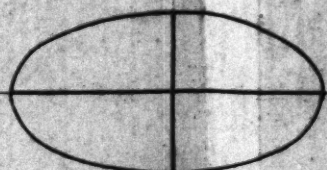
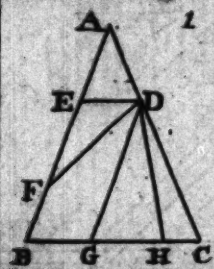
I. A TETRAEDRON, which is a Solid contain'd under four equal and equilateral Triangles.

— Consequently such a Solid is a Pyramid, standing on an equilateral triangular Base, the Superficies whereof will be equal to four times the Area of the Base.

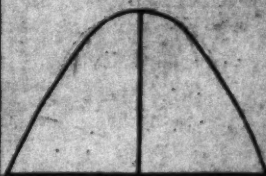
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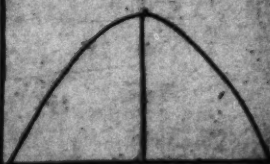




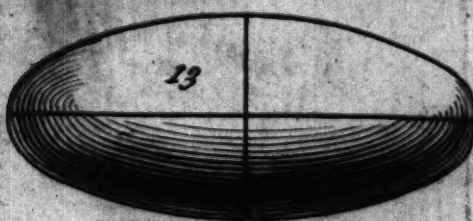
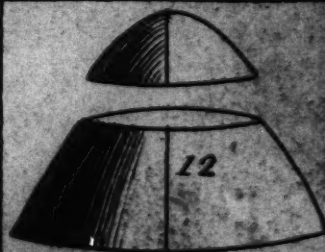
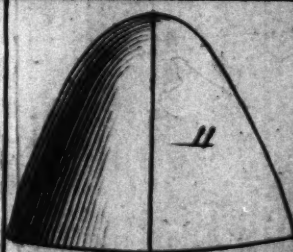
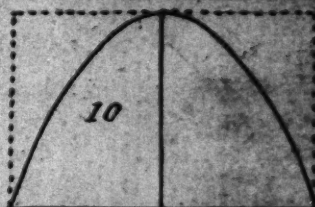
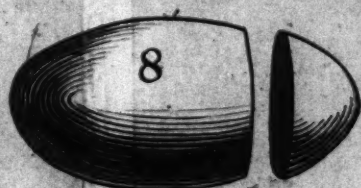
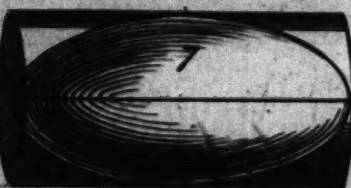
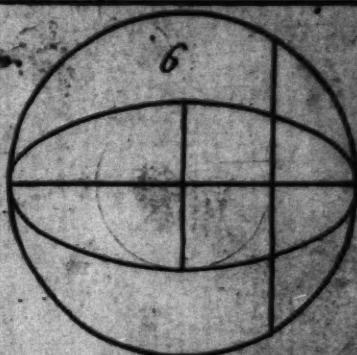
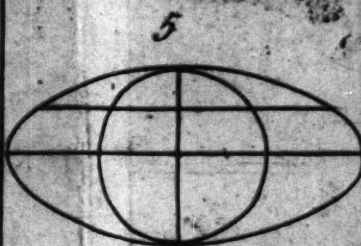
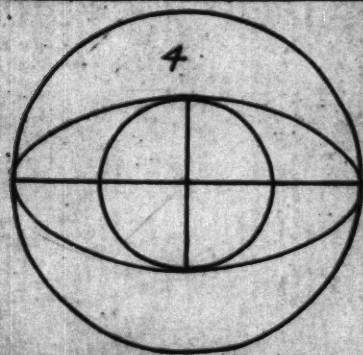
2 Ellipse



3 Parabola



4 Hyperbola



II. AN OCTAEDRON ; which is a Solid, composed of 8 equal Pyramids, whose Vertices all meet in a Point, (which Point is the Center of the Solid) the Base of each being an equilateral Triangle, and equal one to the other.—Therefore the Superficies is equal to eight times the Area of one Triangle, and the Solidity equal to the Solidities of the eight composing Pyramids.

III. A HEXAEDRON or Cube ; which has been already defined.

IV. A DODECAEDRON ; which is a Solid composed of twelve equal Pyramids, whose Vertices all meet in a Point (which Point is the Center of the Solid) the Base being an equilateral Pentagon, and equal to one another.—Therefore the Superficies is equal to twelve times the Area of one Pentagon ; and the Solidity equal to the Solidities of the twelve composing Pyramids.

V. AN ICOSAEDRON ; which is a Solid composed of 20 equal Pyramids, whose Vertices meet all in a Point, (which Point is the Center of the Solid) the Base of each Pyramid being an equilateral Triangle, and equal to one another.—Therefore the Superficies is equal to twenty times the Area of one Triangle ; and the Solidity equal to the Solidities of the twenty composing Pyramids.

BUT because it is somewhat difficult to find the perpendicular Heights of the composing Triangles in each Solid, therefore the following Table is framed that the Solidity and Superficies of any of the regular Solids may be easily attained, by having the Side given.

A TABLE shewing the Superficies and Solidity of any of the regular Solids, whose Side is 1.

| Bodies Names. | Solidity. | Superficies. |
|---------------|-----------|--------------|
| TETRAEDRON. | 0,117851 | 1,732051 |
| OCTAEDRON. | 0,471404 | 3,464102 |
| HEXAEDRON. | 1,000000 | 6,000000 |
| ICOSAEDRON. | 2,181695 | 8,660254 |
| DODECAEDRON. | 7,663119 | 20,645729 |

R U L E I.

MULTIPLY the Square of the given Side, by the Tabular Number under Superficies against the given Name, and the Product is the Superficies.

R U L E II.

MULTIPLY the Cube of the given Side by the Tabular Number under Solidities against the given Name, and the Product is the Solidity.

E X A M P L E.

WHAT is the Solidity and Superficies of an Icofaëdron, one of whose Sides is 40 Inches?

LOOK in the Table under Solidity against the Name *Icofaëdron*, you will find, 2,181695. Then,

$40 \times 40 \times 40 \times 2,181695 = 139628,48 =$ the Solidity of an Icofaëdron, whose Side is 40.

AGAIN

AGAIN, Look in the Table under Superficies, and against the Name *Icosaëdron* you will find,

8,660254. Then $40 \times 40 \times 8,660254 = 13856,4064$, the Superficies; proceed in the same Manner for the other Solids.

Now as all these regular Solids may be inscribed within a Sphere; if your Curiosity should lead you at any Time to know what would be the Sides, Solidities, and Superficies of any of the foremention'd Bodies inscribed in any given Sphere, you may easily satisfy yourself from the following Table and the foregoing one.

A TABLE shewing the Proportions of the Sphere, and the five regular Figures inscribed in the same.

| Bodies Names | Sides. | Superficies. | Solidities. |
|--------------|---------|--------------|-------------|
| TETRAEDRON. | 1,62299 | 4,6188 | 0,15132 |
| HEXAEDRON. | 1,1547 | 8,0000 | 1,5396 |
| OCTAEDRON. | 1,41421 | 6,9282 | 1,3333 |
| DODECAEDRON. | 0,71364 | 10,51462 | 2,78516 |
| ICOSAEDRON. | 1,05146 | 9,57454 | 2,53615 |

The Sphere's Diameter must be 2, to circumscribe these Solids, and such a Sphere's Superficies will be 12,56637, and the Solidity 4,1879.

EXAMPLE.

WHAT will be the Sides, Superficies, and Solidities of the regular Solids inscribed in a Sphere of 12 Inches in Diameter?

$2 : 1,62299 :: 12 : 9,73794 =$ Tetraëdron's Side.

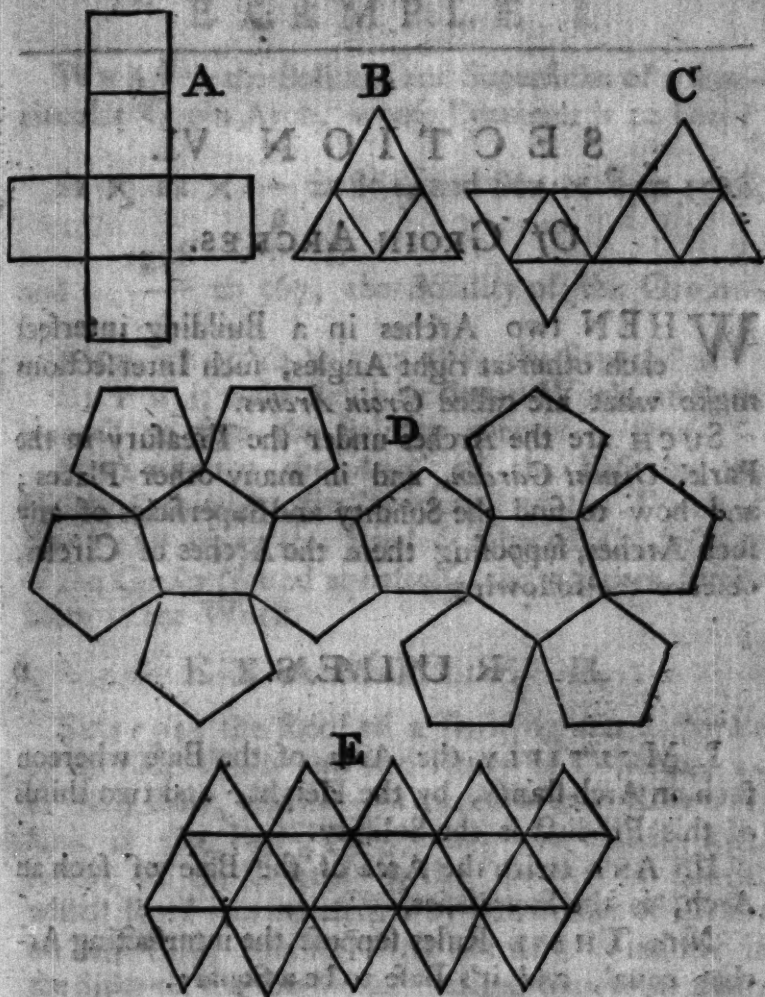
$2 : 1,1547 :: 12 : 6,9282 =$ Hexaëdron's Side.

$2 : 1,41421 :: 12 : 8,48526 =$ Octaëdron's Side.

$2 : 0,71364 :: 12 : 4,28184 =$ Dodecaëdron's Side.

$2 : 1,05146 :: 12 : 6,30876 =$ Icosaëdron's Side.

THUS having got the Length of the Sides, the Solidities and Superficies may easily be found either by this Table or the foregoing one.



If five such Figures as these be cut in Past-board, or any other pliable Matter, and if the Lines be cut half through, and the Sides turned up and glued together, they will represent the regular Solids : viz. A, the Cube or Hexaëdron ; B, the Tetraëdron ; C, the Octaëdron ; D, the Dodecaëdron ; and E, the Icosaëdron.

SECTION VI.

Of GROIN ARCHES.

WHEN two Arches in a Building intersect each other at right Angles, such Intersections make what are called *Groin Arches*.

SUCH are the Arches under the Treasury in the Park, *Cevent Garden*, and in many other Places; and how to find the Solidity and Superficies of any such Arches, supposing them the Arches of Circles, observe the following

R U L E S.

I. MULTIPLY the Area of the Base whereon such an Arch stands, by the Height, and two thirds of this Product is the Solidity.

II. AND twice the Area of the Base of such an Arch, is the Superficies.

Note, THESE Rules suppose the intersecting Arches equal, and it's Base to be a Square.

BUT if the Arches are not equal, or if they be Segments of Ellipses, the Rules for finding the Solidity and Superficies of such Groins, are difficult, and are not easily communicated to a Person not more than an Arithmetician, and therefore I shall not attempt it.

EXAM.

EXAMPLE I.

WHAT is the Solidity and Superficies of a semi-circular Groin Arch, whose Diameter is 12 Feet?

$$12 \times 12 \times \frac{12}{2} = 864, \text{ and } 864 \times 2 = 1728,$$

and $\frac{1728}{3} = 576$, the Solidity of the Groin.

AND, $12 \times 12 \times 2 = 288$, the Superficies.

BUT if it were Brick or Stone Work, and you wanted to find the Solidity thereof; then

MULTIPLY the Area of the Base by the Height [taking in the Thickness of the Work at Top] and from this Product subtract the Solidity of the Groin (found as before) and it leaves the Solidity of the Work.

EXAMPLE II.

SUPPOSE the Roof of a Building was supported by the Side-Walls and two Rows of Columns, 11 in each Row, the two Extrems of each Row were sunk in the End-Walls, and the middle Roof turned off the Columns in the Arch of a Circle, which Roof was intersected by 10 Arches of Circles on each Side, coming from as many Windows in the Sides of the Building, making thereby as many Groin Arches; the Top of the Columns from whence the Arch turned off was square, each Side whereof was 2 Feet; the Chord of each Arch was 20 Feet, and the versed Sine (or Height) 8 Feet: Now supposing the Top of the Roof made flat, by filling up the Spandils, (or Interstices of the Arches without) and the Work continued till it was raised 2 Feet above the Top of the Groin, and supposing

Supposing the Columns were distant from the Sides of the Building 12 Feet; what would be the Solidity and Superficies of this Roof?

First, For the Groins, $20 \times 20 \times 8 = 3200$,
and, $\frac{3200 \times 2}{3} = 2133,8$, the Solidity of one Groat.

AND, $2133,8 \times 10 = 21333,8$, the Solidity of all the Groins.

AGAIN, $20 \times 20 \times 8 + 2 = 4000$, the Solidity of one Parallelopipedon, including one Groat; and $4000 \times 10 = 40000$, the Solidity of all the Parallelopipedons including all the Groins.

THEN, $40000 - 21333,8 = 18666,8$, the Solidity of all the Work above the Groins.

Now each Column being 12 Feet Distance from the Wall, and the Side of the Square on the Top of each Column 2 Feet; therefore the Arch from the Windows to each Groat will be Segments of hollow Cylinders cut parallel to it's Axis, it's Length being 14 Feet; and adding the Lengths of two such together (they being opposite) make 28 Feet: Then finding the Area of the Segment of a Circle, whose Chord is 20 Feet, and versed Sine 8, viz. per PROPOS. XVII. PART I. Meas.

$\frac{20^2}{2} = 10$, and $10 \times 10 = 100$, and $\frac{100}{8} = 12,5$
and $12,5 + 8 = 20,5$, and $\frac{20,5^2}{2} = 10,25$ = to the Radius.

AND, $10,25 - 8 = 2,25$, the Difference.

THEN, $10,25 \times 10,25 \times 14 = 1470,875$, and $10,25 \times 8 \times 2,25 = 184,5$, and $1470,875 - 184,5 = 1286,375$, and $2,25 \times 2,25 \times 6 = 30,375$, and $1286,375 - 30,375 = 1256$, and $10,25 \times 9 = 92,25$, and $2,25 \times 6 = 13,5$, and $9225 + 13,5 = 105,75$.

$= 105,75$, and $\frac{1256}{105,75} = 11,877$, and $11,877 \times \frac{20}{2} = 118,77$, the Area of the Segment.

THIS Area, viz. $118,77 \times 28$, the Length of two opposite cylindrical Segments, gives $3325,56$, the Solidity of one such Segment: And $3325,56 \times 10 = 33255,6$, the Solidity of all the Segments.

AND, $20 \times 8 + 2 \times 28 = 5600$, the Solidity of one rectangular Prism, including one Cylinder's Segment.

AND, $5600 \times 10 = 56000 =$ Solidity of all such Prisms.

AND, $56000 - 33255,6 = 22744,4$, the Solidity of all the Work over the Cylindrical Segments.

AGAIN, $12 + 2 + 20 + 2 + 12 = 48$, the Breadth of the Building.

AND, $48 \times 10 \times 2 = 960$, the Solidity of the Wall separating two Groins, out of which take $118,77 \times 2 = 237,54$, the Solidity of the Segment between two Groins, leaves $722,46$, the Solidity of one parting Wall; and $722,46 \times 10 = 7224,6$, the Solidity of all.

THEN, $18666,6 + 22744,4 + 7224,6 = 48635,6$, the Solidity of the whole Roof.

AGAIN, $20 \times 20 \times 2 \times 10 = 8000$, the Superficies of all the Groins.

AND, $\frac{20}{2} = 10$, and $10 \times 10 = 100$, and $8 \times 8 = 64$, and $100 + 64 = 164$, whose Square Root is $12,8$, per PROP. XV. PART. I. *Meas.* and $\frac{12,8 \times 8 - 20}{3} = 27,46$, the Length of the Arch of one cylindric Segment.

And $27,46 \times 28 \times 10 = 7690,6$, the Superficies of all the Cylindric Segments. AND

AND $12 + 12 = 24$, and $24 \times 2 \times 10 = 480$, the Superficies of the Parts between the cylindric Segments, and the Wall, and Columns.

AND, $27,46 \times 2 \times 10 = 549,2$, the Superficies of the Arches separating the Groins.

THEN $8000 + 7690,8 + 480 + 549,2 = 16720$, the whole Superficies within the Roof.

The following Table shews the specific Gravity or Weight of Metals and other Bodies in Troy Weight and Avoirdupoise Weight.

| | | Oz. Troy. Oz. Avoirdup. | |
|-----------------|-----------------------|----------------------------|-------------|
| | | | |
| A Cubic Inch of | Fine Gold - - | 10,359273 | = 11,365602 |
| | Standard Gold - - | 9,962625 | = 10,930422 |
| | Quicksilver - - | 7,384411 | = 8,101753 |
| | Lead - - - - | 5,984010 | = 6,553885 |
| | Fine Silver - - | 5,850035 | = 6,418324 |
| | Standard Silver - - | 5,556769 | = 6,096596 |
| | Rose Copper - - | 4,747121 | = 5,208369 |
| | Plate Brass - - | 4,404273 | = 4,832116 |
| | Cast Brass - - | 4,272409 | = 4,630300 |
| | Steel - - - - | 4,142127 | = 4,544505 |
| | Common Iron - - | 4,031361 | = 4,422979 |
| | Block-Tin - - | 3,861519 | = 4,236638 |
| | Fine Marble - - | 1,429411 | = 1,568859 |
| | Common Glass - - | 1,360841 | = 1,493037 |
| | Alabaster - - | 0,988456 | = 1,084477 |
| | Dry Ivory - - | 0,962083 | = 1,055542 |
| | Dry Box-Wood - - | 0,543282 | = 0,596057 |
| | Sea Water - - | 0,542742 | = 0,594894 |
| | Common clear ditto | 0,527458 | = 0,578697 |
| | Red Wine - - | 0,523766 | = 0,574646 |
| | Proof Spir. of Brandy | 0,489268 | = 0,536796 |
| | Sound dry Oak - - | 0,489008 | = 0,536569 |
| | Lint-Seed Oyl. - - | 0,491591 | = 0,539345 |
| | Olive Oyl - - | 0,481569 | = 0,528350 |

FROM

FROM this Table it will be very easy to find the Weight of any Body whose Name is mention'd therein, by having the Solidity in cubic Inches given. For multiply the cubic Inches contain'd in any Body (whose Name is in the Table) by the tabular Number, and the Product will be the Weight, either in Troy or Averdupoise, as you take the Number.

EXAMPLE I.

THERE is a Piece of Oak, of a rectangular Form, the Length is 56 Inches, Breadth 18, and Depth 12 Inches; what is the Weight thereof in Averdupoise Weight?

$18 \times 12 \times 56 = 12096$, the solid Inches; and 1 Inch weighs 0,536569 Ounces Averdupoise: Therefore, $12096 \times 0,536569 = 6490,338624$ oz. which is 3 C. 2 grs. 13 lb 10 oz Averdupoise.

HENCE it is very easy to find the Solidity by having the Weight given; for divide any given Weight in Ounces by it's respective tabular Number, and it gives the Solidity in Inches.

IF you would know the Weight of any Body, when immerfed or put into Water, find the Weight of a Quantity of Water equal to the Quantity (or Solidity) of the Body given, and the Weight of Water subtracted from the Weight of the given Body will be it's Weight when immerfed in Water.

EXAM-

EXAMPLE II.

WHAT Weight will sustain a cubic Foot of Iron when put into Water ?

$4,422,979 \times 1728 = 7,642,907,712$, the Weight of a cubic Foot of Iron.

$5,578,697 \times 1728 = 9,630,884,16$, the Weight of a cubic Foot of Water.

THEN, $7,642,907,712 - 9,630,884,16 = 6,679,023,552$ Ounces; and such a Weight of any Matter will sustain a cubic Foot of Iron, when hanging in Water.

THE Solidity of any Body however irregular may be obtained by putting the Body in some Vessel, whose Sides are parallel, and then just cover it with Water, and find the Solidity of that Space the Water occupies, with the Body in it; after which take the Body out, letting the Water drain off: And observing exactly how much the Water has fell; then finding the Solidity of the Space occupied by the Water alone, and the Difference between this and the former Space will be the Solidity of the Body that was immersed.

SECTION VII.

Of the CONSTRUCTION and Use of DECIMAL TABLES.

DECIMAL TABLES ready calculated save much Trouble, and there being frequent Use thereof in this Treatise, and in many more, where any

Mathematical Business is concerned that regards Calculation, I thought it would not be amiss to insert a more compleat Set than as yet I have been able to meet with.

THE Construction of these Tables is performed in the same Manner, as is shewn in the Introduction, SECT. II. Page 8.

CONSTRUCTION of TABLE I.

I. THE Shillings of a Pound Sterling, viz. 19, 18, 17, 16, &c. are separately divided by 20, and the several Quotients are the Decimals of their respective Shillings.

II. THE Decimal of 1 Shilling is divided by 12, (the Pence in a Shilling) and the Quote is the Decimal of a Penny; which Decimal of a Penny multiply'd separately by the several Pence, gives their respective Decimals.

III. THE Decimal of a Penny is divided by 4, (the Farthings in a Penny) and the Quote is the Decimal of a Farthing, which, with multiplying by 2 and 3, gives the other Decimals of Farthings.

CONSTRUCTION of TABLE II.

I. 1 Ounce is divided by 12, (the Number of Ounces in 1 Pound Troy) and the Quotient is the Decimal of 1 Ounce, which multiply'd by the other Ounces, gives their respective Decimals.

U

II. THE

II. THE Decimal of one Ounce is divided by 20, (the Penny Weights in one Ounce) and the Quote is the Decimal of 1 Penny Weight, which multiply'd by the other Penny Weights gives their respective Decimals. And from these are the Decimals of Grains, constructed in the same Manner.

CONSTRUCTION of TABLE III.

I. BECAUSE 20 Hundred is one Ton ; therefore the Decimals of the Shillings will serve for the Hundred Weights, supposing 1 Ton the Integer.

II. THE Decimal of 1 Shilling (*i. e.* 1 Hundred Weight) is divided by 4 (the Quarters in 1 Hundred Weight) and the Quote is the Decimal of 1 Quarter, from whence the Decimals of the other Quarters are obtained, as are also the Decimals of Pounds.

HUNDRED WEIGHT the INTEGER.

THE Decimals of the Quarters are ,25 ,5 and ,75, and dividing the Decimal of 1 Quarter by 28, (the Pounds in a Quarter) gives the Decimal of 1 Pound ; from whence the Decimals of the other Pounds are obtained, as are also the Decimals of Ounces ; and from them the Decimals of Drams.

ONE

ONE POUND *the* INTEGER.

1 OUNCE is divided by 16, (the Ounces in 1 Pound) and it gives the Decimal of 1 Ounce, from whence the Decimals of the other Ounces are obtained, as are also the Decimals of Drams.

AFTER the same Manner are the other Tables of Measures and Time constructed, having always a due Regard how many of a lesser Denomination are contained in a superiour one.

IN the Construction of these Tables there are some little Liberties taken, which is, As the Tables are only carry'd to 6 Places, (excepting in some Particulars) whenever the seventh Figure would have been more than a 5, the sixth Place has been increased with Unity, otherwise, the sixth Place is given as it turns out in the Operation. And universally, the last Place is always increased with Unity, if the next succeeding Place would be more than a 5.

THE Use of these Tables are obvious, but to prevent all Doubts, I will here insert two or three.

EXAMPLE I.

WHAT is the Decimal of 7 oz. 16 dwt, 18 grs.
In TABLE II.

against 7 oz. is ,5833333

against 16 dwt. is ,0866666

against 18 grs. is ,0031250

These added, their Sum is ,653125, the Decimal required.

EXAMPLE

EXAMPLE II.

WHAT is the Decimal of 46 Gallons and 5 Pints?

In TABLE IV.

Against 40 Gallons is ,634920

Against 6 Gallons is ,395238

Against 5 Pints is ,009921

These added, their Sum is ,740079, the Decimal required.

AFTER the same Manner are Decimals of other enominations collected.

N.B. If you find not the Number you want in any of the Tables, take it out at twice, thus: For 17 *dwt.* add 10 and 7 together, and the like for any other.



Printed by C. Jephson in West-Smithfield.

TABLE I.
Coin, 1 to the Integer.

| S. | Dec. | S. | Dec. |
|----|------|----|------|
| 19 | ,95 | 9 | ,45 |
| 18 | ,9 | 8 | ,4 |
| 17 | ,85 | 7 | ,35 |
| 16 | ,8 | 6 | ,3 |
| 15 | ,75 | 5 | ,25 |
| 14 | ,7 | 4 | ,2 |
| 13 | ,65 | 3 | ,15 |
| 12 | ,6 | 2 | ,1 |
| 11 | ,55 | 1 | ,05 |
| 10 | ,5 | | |

| Pence. | Decimals. |
|--------|-------------|
| 11 | ,04583 . |
| 10 | ,0416 . . |
| 9 | ,0375 . . |
| 8 | ,03 |
| 7 | ,02916 . |
| 6 | ,025 . . . |
| 5 | ,02083 . |
| 4 | ,016 . . . |
| 3 | ,0125 . . |
| 2 | ,0013 . . |
| 1 | ,00416 . |

| Farthings. | Decimals. |
|------------|-----------|
| 3 | ,003125 |
| 2 | ,002083 |
| 1 | ,0010416 |

Note. The following Table of Ounces will serve for Inches and any thing, where 12 is the Integer.

TABLE II.
Troy Weight.
1 to the Integer.

| Ounces. | Decimals. |
|---------|---------------|
| 11 | ,916 . . . |
| 10 | ,88 |
| 9 | ,75 |
| 8 | ,66 |
| 7 | ,583 . . . |
| 6 | ,5 |
| 5 | ,416 . . . |
| 4 | ,33 |
| 3 | ,25 |
| 2 | ,16 |
| 1 | ,083 |

| Dwts. | Decimals. |
|-------|-------------|
| 10 | ,0416 . . |
| 9 | ,0375 . . |
| 8 | ,03 |
| 7 | ,02916 . |
| 6 | ,025 . . . |
| 5 | ,02083 . |
| 4 | ,016 . . . |
| 3 | ,0125 . . |
| 2 | ,0083 . . |
| 1 | ,00416 . |

| Gr. | 10 | Decimals. |
|-----|----|-----------|
| | 9 | ,001563 |
| | 8 | ,001389 |
| | 7 | ,001215 |
| | 6 | ,001042 |
| | 5 | ,000868 |
| | 4 | ,000694 |
| | 3 | ,000521 |
| | 2 | ,000347 |
| | 1 | ,000173 |

TABLE III.
Avoirdup. 1 Ton the Integ.

| <i>Qrs. Cws.</i> | <i>Decimals.</i> |
|------------------|------------------|
| 3 | ,0375 .. |
| 2 | ,025 ... |
| 1 | ,0125 .. |

| <i>Pounds.</i> | <i>Decimals.</i> |
|----------------|------------------|
| 20 | ,008928 |
| 10 | ,004464 |
| 9 | ,004018 |
| 8 | ,003571 |
| 7 | ,003125 |
| 6 | ,002678 |
| 5 | ,002232 |
| 4 | ,001785 |
| 3 | ,001339 |
| 2 | ,000892 |
| 1 | ,000446 |

1 Pound the Integer.

| <i>Ounces.</i> | <i>Decimals.</i> |
|----------------|------------------|
| 10 | ,625 ... |
| 9 | ,5625 .. |
| 8 | ,5... .. |
| 7 | ,4375 .. |
| 6 | ,375 ... |
| 5 | ,3125 .. |
| 4 | ,25 ... |
| 3 | ,1875 .. |
| 2 | ,125 ... |
| 1 | ,0625 .. |

TABLE III.
Avoirdup. 1 lb the Integ.

| <i>Drams.</i> | <i>Decimals.</i> |
|---------------|------------------|
| 10 | ,039062 |
| 9 | ,035156 |
| 8 | ,031250 |
| 7 | ,027344 |
| 6 | ,023437 |
| 5 | ,019531 |
| 4 | ,015625 |
| 3 | ,011718 |
| 2 | ,007812 |
| 1 | ,003906 |

1 Cwt. the Integer.

| <i>Pounds.</i> | <i>Decimals.</i> |
|----------------|------------------|
| 20 | ,178571 |
| 10 | ,089285 |
| 9 | ,080357 |
| 8 | ,071428 |
| 7 | ,0625 |
| 6 | ,053571 |
| 5 | ,044642 |
| 4 | ,035714 |
| 3 | ,026785 |
| 2 | ,017857 |
| 1 | ,008928 |

| <i>Ounces.</i> | <i>Decimals.</i> |
|----------------|------------------|
| 10 | ,005580 |
| 9 | ,005022 |
| 8 | ,004464 |
| 7 | ,003906 |
| 6 | ,003348 |
| 5 | ,002790 |
| 4 | ,002232 |
| 3 | ,001674 |
| 2 | ,001116 |
| 1 | ,000558 |

TABLE III.

Avoirdupoise.
1 Cwt. the Integer.

| Drams. | Decimals. |
|--------|-----------|
| 10 | ,000349 |
| 9 | ,000314 |
| 8 | ,000279 |
| 7 | ,000244 |
| 6 | ,000209 |
| 5 | ,000174 |
| 4 | ,000139 |
| 3 | ,000104 |
| 2 | ,000069 |
| 1 | ,000034 |

TABLE IV.

Liquid Measure.
1 Tun the Integer.

| Gallons. | Decimals. |
|----------|-----------|
| 200 | ,793651 |
| 100 | ,396825 |
| 90 | ,357141 |
| 80 | ,317460 |
| 70 | ,277... . |
| 60 | ,238095 |
| 50 | ,198412 |
| 40 | ,158730 |
| 30 | ,119047 |
| 20 | ,079365 |
| 10 | ,039682 |
| 9 | ,035714 |
| 8 | ,031746 |
| 7 | ,027... . |
| 6 | ,023809 |
| 5 | ,019841 |
| 4 | ,015873 |
| 3 | ,011904 |
| 2 | ,007936 |
| 1 | ,003968 |

TABLE V.

Liquid Measure.
1 Tun the Integer.

| Pints. | Decimals. |
|--------|-----------|
| 7 | ,003472 |
| 6 | ,002976 |
| 5 | ,002480 |
| 4 | ,001984 |
| 3 | ,001488 |
| 2 | ,000992 |
| 1 | ,000496 |

1 Hoghead the Integer.

| Gallons. | Decimals. |
|----------|-----------|
| 60 | ,952381 |
| 50 | ,793651 |
| 40 | ,634921 |
| 30 | ,476190 |
| 20 | ,317460 |
| 10 | ,158730 |
| 9 | ,142857 |
| 8 | ,126984 |
| 7 | ,111111 |
| 6 | ,095238 |
| 5 | ,079365 |
| 4 | ,063492 |
| 3 | ,047619 |
| 2 | ,031746 |
| 1 | ,015873 |

| Pints. | Decimals. |
|--------|-----------|
| 7 | ,013889 |
| 6 | ,011905 |
| 5 | ,009921 |
| 4 | ,007937 |
| 3 | ,005952 |
| 2 | ,003968 |
| 1 | ,001984 |

TABLE V.

*Long Measure.**1 Mile the Integer.*

| <i>Yards.</i> | <i>Decimals.</i> |
|---------------|------------------|
| 1000 | ,568182 |
| 900 | ,511364 |
| 800 | ,454545 |
| 700 | ,397727 |
| 600 | ,340909 |
| 500 | ,284091 |
| 400 | ,227272 |
| 300 | ,170454 |
| 200 | ,113636 |
| 100 | ,056818 |
| 90 | ,051136 |
| 80 | ,045454 |
| 70 | ,039772 |
| 60 | ,034090 |
| 50 | ,028409 |
| 40 | ,022727 |
| 30 | ,017045 |
| 20 | ,011363 |
| 10 | ,005681 |
| 9 | ,005113 |
| 8 | ,004545 |
| 7 | ,003977 |
| 6 | ,003409 |
| 5 | ,002841 |
| 4 | ,002273 |
| 3 | ,001704 |
| 2 | ,001136 |
| 1 | ,000568 |

TABLE V.

*Long Measure.**1 Mile the Integer.*

| <i>Fest.</i> | <i>Decimals.</i> |
|----------------------------|------------------|
| 2 | ,0003787 |
| 1 | ,0001894 |
| <i>Inches.</i> | <i>Decimals.</i> |
| 9 | ,0001421 |
| 6 | ,0000947 |
| 3 | ,0000474 |
| 1 | ,0000158 |
| <i>1 Yard the Integer.</i> | |
| <i>Fest.</i> | <i>Decimals.</i> |
| 2 | 6 |
| 1 | 3 |
| <i>Inches.</i> | <i>Decimals.</i> |
| 11 | ,308 |
| 10 | ,27 |
| 9 | ,248 |
| 8 | ,21 |
| 7 | ,194 |
| 6 | ,16 |
| 5 | ,138 |
| 4 | ,11 |
| 3 | ,083 |
| 2 | ,05 |
| 1 | ,027 |
| <i>Qrs. Inch.</i> | <i>Decimals.</i> |
| 3 | ,02083 . |
| 2 | ,0138 . . |
| 1 | ,00694 . |

TABLE VI.

Time.

1 Year the Integer.

| Days. | Decimals. |
|-------|-----------|
| 300 | ,821918 |
| 200 | ,547945 |
| 100 | ,273973 |
| 90 | ,246575 |
| 80 | ,219178 |
| 70 | ,191781 |
| 60 | ,164383 |
| 50 | ,136986 |
| 40 | ,109589 |
| 30 | ,082192 |
| 20 | ,054792 |
| 10 | ,027397 |
| 9 | ,024657 |
| 8 | ,021917 |
| 7 | ,019178 |
| 6 | ,016438 |
| 5 | ,013698 |
| 4 | ,010959 |
| 3 | ,008219 |
| 2 | ,005479 |
| 1 | ,002740 |

TABLE VI.

Time.

1 Year the Integer.

| Hours. | Decimals. |
|--------|-----------|
| 20 | ,002282 |
| 11 | ,001141 |
| 9 | ,001027 |
| 8 | ,000913 |
| 7 | ,000799 |
| 6 | ,000685 |
| 5 | ,000571 |
| 4 | ,000456 |
| 3 | ,000342 |
| 2 | ,000228 |
| 1 | ,000114 |

Minutes. Decimals.

| | |
|----|-----------|
| 50 | ,000951 |
| 40 | ,0000761 |
| 30 | ,00005706 |
| 20 | ,00003804 |
| 10 | ,00001902 |
| 9 | ,00001711 |
| 8 | ,00001521 |
| 7 | ,00001331 |
| 6 | ,00001141 |
| 5 | ,00000951 |
| 4 | ,00000761 |
| 3 | ,00000571 |
| 2 | ,00000380 |
| 1 | ,00000190 |

Qrs. Min. Decimals.

| | |
|---|-----------|
| 3 | ,00000142 |
| 2 | ,00000095 |
| 1 | ,00000047 |

TABLE VI.

*Time.**1 Day the Integer.*

| <i>Hours.</i> | <i>Decimals.</i> |
|-----------------|------------------|
| 20 | ,82 |
| 10 | ,416 . . . |
| 9 | ,375 . . . |
| 8 | ,3 |
| 7 | ,2916 . . |
| 6 | ,25 |
| 5 | ,2083 . . |
| 4 | ,16 |
| 3 | ,125 . . . |
| 2 | ,083 . . . |
| 1 | ,0416 . . |
| <i>Minutes.</i> | <i>Decimals.</i> |
| 50 | ,03472 |
| 40 | ,027 . . . |
| 30 | ,02083 . |
| 20 | ,0138 . . |
| 10 | ,00694 . |
| 9 | ,00625 . |
| 8 | ,005 . . |
| 7 | ,004861 |
| 6 | ,00416 . |
| 5 | ,003472 |
| 4 | ,0027 . . |
| 3 | ,002083 |
| 2 | ,00138 . |
| 1 | ,000694 |
| <i>Seconds.</i> | <i>Decimals.</i> |
| 45 | ,0005208 |
| 30 | ,0003472 |
| 15 | ,0001736 |

TABLE VII.

*Various Measures.**1 ft. Cloth Measures.*
1 Yard the Integer.

| <i>Yrs. Yard.</i> | <i>Decimals.</i> |
|-------------------|------------------|
| 3 | ,75 |
| 2 | ,5 . |
| 1 | ,25 |
| <i>Nails.</i> | <i>Decimals.</i> |
| 3 | ,1875 |
| 2 | ,125 . |
| 1 | ,0625 |

*Measure.**Liquid. | Dry.**Integer.**1 Gall. | 1 Quartor.*

| <i>Pints.</i> | <i>Decim.</i> | <i>Bushe.</i> |
|---------------|---------------|---------------|
| 7 | ,875 | 7 |
| 6 | ,76 . | 6 |
| 5 | ,625 | 5 |
| 4 | ,5 . . | 4 |
| 3 | ,375 | 3 |
| 2 | ,25 . | 2 |
| 1 | ,125 | 1 |

| <i>Qs. Ps.</i> | <i>Decim.</i> | <i>Pecks.</i> |
|----------------|---------------|---------------|
| 3 | ,09375 | 3 |
| 2 | ,0625 . | 2 |
| 1 | ,03125 | 1 |

| <i>Decimals.</i> | <i>Qrs. Pecks.</i> |
|------------------|--------------------|
| ,023437 | 3 |
| ,015625 | 2 |
| ,007812 | 1 |

| <i>Decimals.</i> | <i>Pints.</i> |
|------------------|---------------|
| ,005859 | 3 |
| ,003906 | 2 |
| ,001953 | 1 |

ERRATA.

In the Advertisement, *r.* Optician.

Page 7, the Quote should be ,0084971334, &c.

P. 19. EXAM. I. the Product by $\frac{1}{2}$, is 9279408.

P. 44, l. last but one, *dele* subtracted.

P. 124, PROP. XVII. in the Rule, l. 1. *r.* of the Circle l. 2, *dele* a.

P. 128, In finding the Segments Areas in this Ex. there is an Omission of Multiplying by the half

Chord; therefore P. 129. l. 5. *r.* $2,56 \times \frac{6}{2}$

$= 7,68$, the Area; and l. 18. *r.* $2,2 \times \frac{4}{2} = 4,4$

the Area. Then $21,75 + 12 + 7,68 + 4,4$
 $= 45,83$ Square Chains. l. 21. *r.* 4 Acres, 5
 Square Chains, 13 Square Poles, &c.

P. 144, l. 20. *r.* add the several, &c.

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